

# A Physical Model of the Energy Flow in Space Habitats

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## Abstract

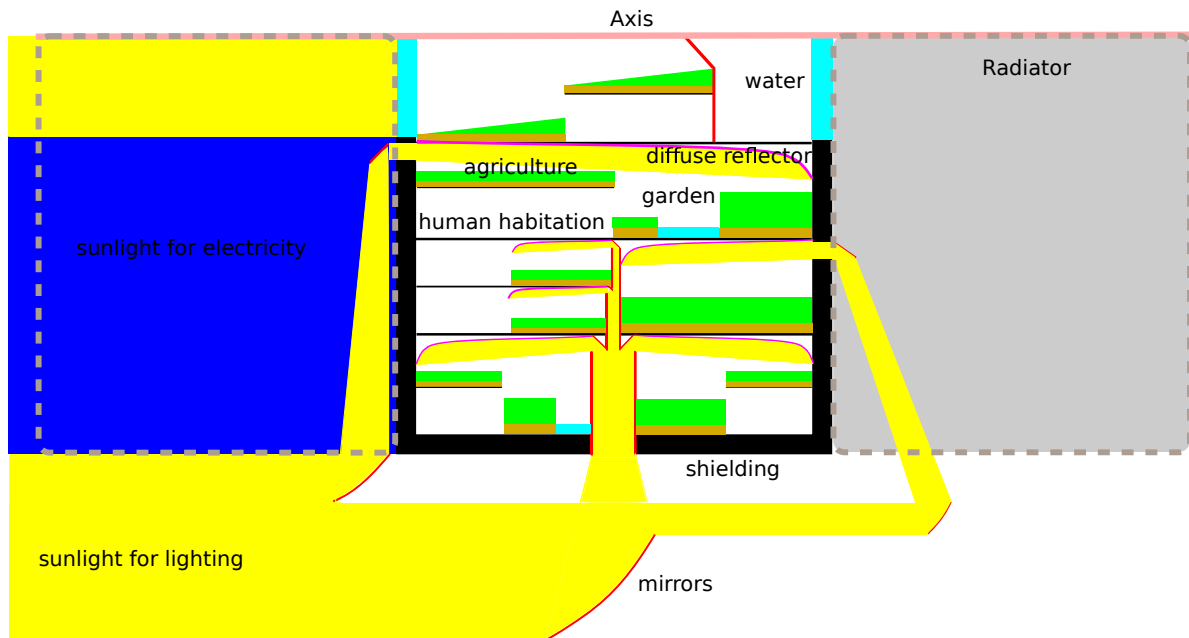
Space habitats as free-floating ecosystems require supply of light and electricity and removal of waste heat. This paper aims to estimate size and mass of cooling, lighting, and electricity systems. A scalable physical model is developed that assumes cylindrical space habitats with constant power per volume (so the habitat power is used as a proxy for size). Solar energy covers both electrical and light demand, the latter by concentration and distribution through light channels. Heat is dissipated through the hull and by circulation of a coolant between the habitat interior and radiators. The coolant can be a liquid, the habitat air, or it evaporates during heat absorption and condenses in the radiator. The model allows an approximate optimization of the power to counter friction in the cooling system. Both lighting and cooling are more challenging for larger habitats as the distances between interior and mirrors or radiators increase. For default parameters, heat transmission through the hull suffices only up to 4kW of habitat power, and unconcentrated lighting through a transparent hull up to 500kW. A limit for cooling is reached around  $10^{13}$ W, where liquid coolant would start dominating total mass, vapor would need too much volume, and air would require too much fan power. Also at this habitat size, light channels would take up too much interior volume, demanding electric lighting. Since the burden of shielding mass decreases with surface per volume, a region of low mass per power is found at around  $10^{11}$ W (on the order of a million inhabitants). At this size, the cooling system mass is estimated at  $0.12 \frac{\text{kg}}{\text{W}}$  for liquid, 0.05 for vapor, and 0.03 for air, while lighting amounts to 0.003 and electricity to 0.005 - much less than interior (assumed at 2.5) and shielding (0.7).

## 1 Introduction

All known life is still confined to the deep gravity well of Earth. However, human civilization has the unique chance of spreading life to space by creating closed ecosystems on other planetary bodies or within free-floating space habitats. The constant sunlight in the latter case provides easier access to energy for photosynthesis and electricity, but matter has to be brought from somewhere, e.g. asteroids [1]. Once built, matter cycles within the ecosystem, driven by the dissipation of energy from sunlight to heat, which has to be removed from the habitat.

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**Figure 1:** Example of a small, scalable habitat (Sun to the left, cylindrical symmetry). Electricity is produced by PV panels on the front. Sunlight enters the habitat directly through a water-filled window or via external concentrating mirrors (concentration can be much higher than shown). A circulating coolant (not shown) takes the waste heat to two sheets of radiators. With a power of 1MW, this cylindrical habitat of about 20m radius (divided in 4 floors) can accommodate 25 people. Human habitation and agriculture each take up a quarter of the volume, leaving a half for garden/wildlife.

Early design studies of space habitats were done by O’Neill and others in the 1970s [8, 7]. High launch costs have since prevented their realization, but it has also been proposed to start with much smaller and lighter habitats in low Earth orbit [4]. A review can be found in [2]. Cooling methods have not yet been computed in detail.

This article presents a physical model of the energy flow in space habitats. Sec. 2 describes the model, whose results are presented in Sec. 3. The set of equations is derived in the appendix (Secs. A for light and electricity supply, and B for cooling).

The model is implemented as a python program, publicly available on github (<https://github.com/RainerRolffs/SpaceHabitats>).

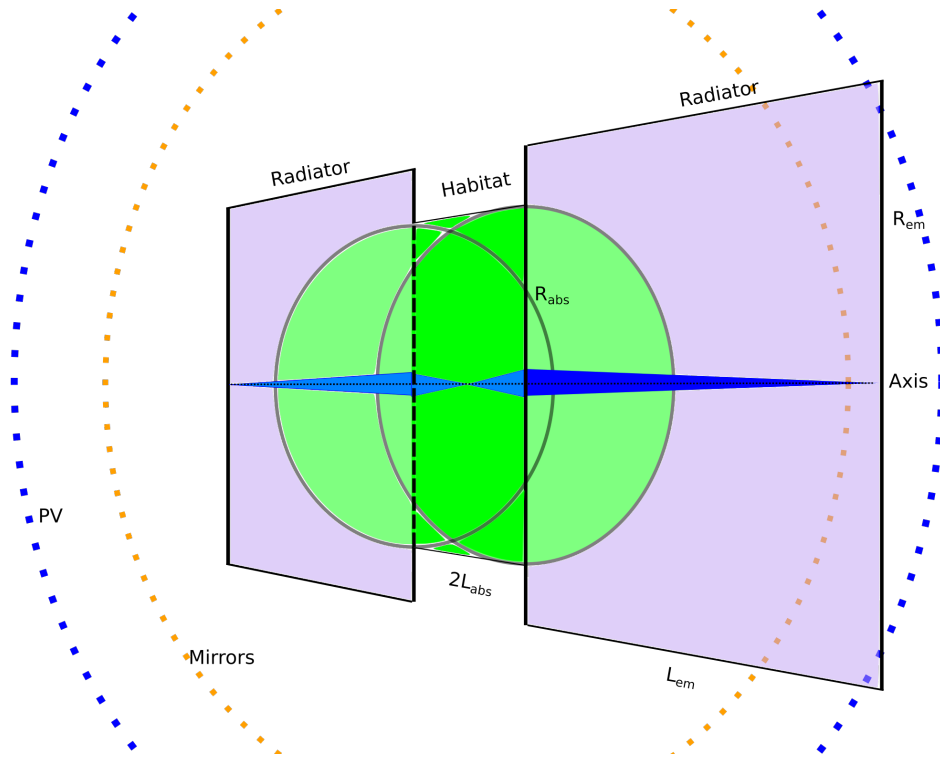
## 2 Model Description

The model assumes a space habitat of constant power per volume and cylindrical shape (Sec. 2.1), which is provided with light and electricity by mirrors and photovoltaics (Sec. 2.2). This heat is dissipated by hull transmission (Sec. 2.3) and by circulating liquid, air, or vapor coolant (Sec. 2.4). The coolant absorbs the habitat’s waste heat (Sec. 2.5) and transports it to the radiator, where it is emitted (Sec. 2.6). Additional electric power is required to compensate friction losses (Sec. 2.7).

Figs. 1 and 2 depict the structure of habitat and cooling system. Table 1 summarizes the input parameters and their default values.

**Table 1: Input Parameters**

Description	Appendix Symbol	Program Variable	Unit	Default Value
<b>Habitat</b>				
power consumed by the habitat	$P_H$	powers	W	$10^3 - 10^{18}$
habitat power density	$\nu_P$	powerPerVolume	$\frac{W}{m^3}$	25
habitat interior mass per power	$\mu_H$	interiorMassPerPower	$\frac{kg}{W}$	2.5
cylinder length to radius	$\xi$	aspectRatio		1.3
fraction of habitat power inside the shielding	$f_{P,in}$	insidePowerFraction		1
<b>Energy Collection</b>				
distance to the Sun	$d_{\odot}$	solarDistance	AU	1
fraction of shaded sunlight	$f_{sh}$	shadedFraction		0
electric fraction of the habitat power	$f_E$	electricFraction		0.25
efficiency of converting sunlight to electricity	$\eta_E$	electricEfficiency		0.2
surface density of PV modules	$\sigma_E$	electricSurfaceDensity	$\frac{kg}{m^2}$	5
concentration factor	$\gamma$	concentrationFactor		400
reflectivity of outer mirrors	$r_{M,out}$	outerReflectivity		0.5
reflectivity of windows	$r_W$	windowReflectivity		0.3
absorptivity of windows	$a_W$	windowAbsorptivity		0.05
maximum window temperature	$T_{W,max}$	maxWindowTemperature	K	500
reflectivity of channel mirrors	$r_{M,in}$	innerReflectivity		0.99
channel surface intensity	$I_{  }$	surfaceIntensity	$\frac{W}{m^2}$	500
average surface density of light collection	$\sigma_L$	lightSurfaceDensity	$\frac{kg}{m^2}$	1
max. fraction of habitat volume occupied by light channels	$f_{L,max}$	maxLightVolumeFraction		0.2
<b>Hull</b>				
shielding surface density	$\sigma_S$	hullSurfaceDensity	$\frac{kg}{m^2}$	5000
hull density	$\rho_S$	hullDensity	$\frac{kg}{m^3}$	1000
hull conductivity	$\lambda_S$	hullConductivity	$\frac{W}{Km}$	1
absorptivity of outer hull surface	$a_S$	hullSurfaceAbsorptivity		0
gap thickness	$d_{gap}$	gapThickness	m	0.1
gap location as fraction of hull from the inside	$f_{gap}$	gapLocation		0.1
emissivity of inner gap surface	$\epsilon_{gap,in}$	innerGapEmissivity		0.9
emissivity of outer gap surface	$\epsilon_{gap,out}$	outerGapEmissivity		0.9
gap convective heat transfer coefficient	$\alpha_{gap}$	gapTransferCoeff	$\frac{W}{Km^2}$	5
conductivity within the gap	$\lambda_{gap}$	gapConductivity	$\frac{W}{Km}$	0.01
<b>Coolant</b>				
type of coolant		coolantType	<i>Liquid, Vapor, Air</i>	<i>Liquid</i>
density of the liquid coolant	$\rho_{C,L}$	liquidDensity	$\frac{kg}{m^3}$	$10^3$
heat capacity of the liquid coolant	$c_{C,L}$	liquidHeatCapacity	$\frac{J}{kgK}$	4280
latent heat of vapor	$\phi$	vaporLatentHeat	$\frac{J}{kg}$	$2.453 \cdot 10^6$
<b>Habitat Temperature</b>				
minimum habitat temperature	$T_{H,min}$	minHabTemp	K	273+12
maximum habitat temperature	$T_{H,max}$	maxHabTemp	K	273+27
<b>Heat Absorption (Liquid and Vapor)</b>				
temperature difference between leaving and entering flow	$\delta T_{abs}$	tempDiffFlow	K	20
minimum temperature difference between habitat and coolant temperature	$\delta T_{HC}$	minTempDiffHabCoolant	K	5
heat transfer coefficient between habitat air and coolant	$\alpha_{abs}$	absorptionTransferCoeff	$\frac{W}{Km^2}$	20
absorption pipe surface density	$\sigma_{abs}$	absorptionSurfaceDensity	$\frac{kg}{m^2}$	3
max. fraction of the habitat volume occupied by coolant	$f_{C,max}$	maxCoolantVolumeFraction		0.2
<b>Heat Absorption (Air)</b>				
relative humidity of outgoing air	$h_{r,out}$	outgoingRelativeHumidity		0.8
air pressure	$p_A$	airPressure	bar	0.4
inner surface per habitat power	$\psi$	innerSurfacePerPower	$\frac{m^2}{W}$	0.01
fraction of the habitat volume occupied by airflow	$f_A$	windyVolumeFraction		0.25
<b>Heat Emission</b>				
radiator emissivity	$\epsilon$	emissivity		0.9
average sky temperature	$T_{sky}$	skyTemp	K	3
radiator surface density	$\sigma_{em}$	emissionSurfaceDensity	$\frac{kg}{m^2}$	5
maximum ratio of radiator to habitat radius	$\kappa_{em}$	maxRadiatorToHabRadius		2
<b>Friction</b>				
efficiency of pump or fan	$\eta_F$	pumpEfficiency		0.8
assumed minimum friction factor at very high Reynolds	$\lambda_{min}$	minFrictionFactor		0.005
if friction is optimized	$b_{opt}$	isFrictionOptimized		true
(initial) absorption friction per habitat power (not for Air)	$\chi_{abs,st}$	absorptionFrictionFraction		0.01
(initial) connection friction per habitat power	$\chi_{con,st}$	connectionFrictionFraction		0.01
(initial) emission friction per habitat power	$\chi_{em,st}$	emissionFrictionFraction		0.01
max. total friction per habitat power	$\chi_{max}$	maxFrictionFraction		0.5



**Figure 2:** Sketch of the cooling system, consisting of two symmetric parts. In each part, heat is absorbed in a cylindrical volume of radius  $R_{abs}$  and length  $L_{abs}$  (green) by a coolant flowing from the axis outwards and back (perpendicular to the axis in all directions). The coolant is transported outside through a connection pipe along the axis (blue), whose cross section varies as part of the coolant returns when it has emitted or absorbed its share of heat (the pipe's length is  $L_{abs}$  inside and  $L_{em}$  outside the habitat). From the outer connection pipe, the coolant flows a distance  $R_{em}$  and back, thereby emitting the heat from both sides of the planar radiator sheet (gray). The shown proportions match a default habitat of  $10^8\text{W}$  ( $R_{abs} = 100\text{m}$ ,  $R_{em} = 145\text{m}$ ). Mirrors for light collection and PV modules extend out to a radius of  $300\text{m}$ . The radii and lengths of heat absorption scale with the cube root of power, those of heat emission and energy collection with the square root.

## 2.1 Habitat

The habitat power  $P_H$  is the total energy flow through the habitat (electricity and light). By default, a range from  $10^3$  to  $10^{18}\text{W}$  is computed (100 runs with logarithmic spacing).

To fully use the precious shielded volume, the habitat interior can be divided in floors (or spiraling ground) of different heights and gravity levels (Fig. 1). This diversity of gravity can be an advantage compared to the single value on Earth, as lower gravity would be pleasant for humans and other life forms. Also, half of a cylindrical volume is at above 70% of the maximum (and more of the ground area if floors are higher at lower gravity), and usage would be adapted to gravity. With such stacked landscapes, the interior volume is assumed to be proportional to the habitat power  $P_H$  with power density (power per volume)  $\nu_P$ . In this way, the habitat power stands for the interior volume as well as the number of inhabitants. For example, if each person needs  $100\text{m}^2$  for each garden (8m height), agriculture (4m), and human habitation (4m), this yields a volume of  $1600\text{m}^3$  per person. A demand of  $10\text{kW}$  electricity and  $30\text{kW}$  light (average irradiation of  $150\frac{\text{W}}{\text{m}^2}$  on garden and agriculture) results in a habitat power of  $40\text{kW}$  per person, so the default value of the power density is  $40\text{kW}$  in  $1600\text{m}^3$ , or  $25\frac{\text{W}}{\text{m}^3}$ .

Interior mass per power defaults to  $2.5 \frac{\text{kg}}{\text{W}}$  (equivalent to half a meter of water per 8m of floor height, so  $500 \frac{\text{kg}}{\text{m}^2}$  per  $200 \frac{\text{W}}{\text{m}^2}$ ). This includes constructions, soil, water, air, and life, but is only used for comparison. The cylinder aspect ratio  $\xi$  (length to radius) is 1.3, so the largest moment of inertia is around the axis for rotational stability [3], and the volume per surface is not much less than that of the sphere.

Part of the habitat power may be used outside the shielding, e.g. greenhouses combined with air flow cooling [6], so the power fraction inside the shielding can be specified as  $f_{P,\text{in}}$  (default 1).

## 2.2 Energy Collection

The energy collection system provides the habitat with both electricity and light through photovoltaics and mirrors that concentrate sunlight on windows, which pass it to light channels for distribution.

The power density of sunlight depends on the distance to the Sun  $d_{\odot}$  (default is Earth location at 1AU). Obstacles between the Sun and the habitat are incorporated as the fraction of shaded sunlight  $f_{\text{sh}}$  (default 0).

The fraction of the habitat power that is needed as electricity is  $f_E$  (default a quarter), the rest is (mostly visible) light. The efficiency of converting sunlight to electricity  $\eta_E$  defaults to 0.2, and the surface density of PV modules  $\sigma_E$  to  $5 \frac{\text{kg}}{\text{m}^2}$ . In perpetual sunlight, this system produces  $54 \frac{\text{W}}{\text{kg}}$  ( $1360 \frac{\text{W}}{\text{m}^2}$  times  $\frac{\eta_E}{\sigma_E}$ ).

Sunlight is concentrated on windows by a factor  $\gamma$  (default is 400, so 20-fold concentration in both dimensions and about  $5^\circ$  angular deviation). Undesired wavelengths are filtered out, either at the outer mirrors (reflectivity  $r_{M,\text{out}}$  of 0.5), or at the windows (reflectivity  $r_W$  of 0.3). As the windows absorb a fraction  $a_W$  (default 0.05), they have to be cooled if their temperature exceeds the maximum  $T_{W,\text{max}}$  (default 500K).

The light channel walls should be as reflective as possible, aided by the flat incident angles ( $r_{M,\text{in}}$  defaults to 0.99). They leak an average intensity of  $I_{\parallel}$  to the habitat (default  $500 \frac{\text{W}}{\text{m}^2}$ ) as the channels are tapped by moving mirrors inside.

The mirror surface density  $\sigma_L$  (of  $1 \frac{\text{kg}}{\text{m}^2}$ ) is applied to the projected outer mirror surface and the light channel walls (windows count as hull, since shielding would be rearranged). Both surface densities are conservative as PV modules and mirrors could be made even thinner, and focusing sunlight on PV would help especially at large solar distances.

A size limit for light distribution is reached when channels take up too much interior volume (maximum fraction  $f_{L,\text{max}}$  defaults to 0.2). The remaining lighting demand is then replaced by electricity.

## 2.3 Hull

The shielding surface density  $\sigma_S$  defaults to  $5,000 \frac{\text{kg}}{\text{m}^2}$ . Additional protection can come from a natural or artificial magnetic field, from a low orbit around a planetary body, or from interior shielding. At 500km above Earth's equator, even no shielding would be possible, while in deep space  $7,000 \frac{\text{kg}}{\text{m}^2}$  might be necessary [5].

Further hull parameters are required for the hull heat transfer computation. Shielding density  $\rho_S$  and thermal conductivity  $\lambda_S$  default to  $1000 \frac{\text{kg}}{\text{m}^3}$  and  $1 \frac{\text{W}}{\text{Km}}$ . The surface absorptivity of sunlight  $a_S$  defaults to 0 as the hull is shaded by PV.

The hull consists of two air-tight envelopes with reduced pressure between them. This gap of thickness  $d_{\text{gap}}$  (0.1m) is located at a fraction  $f_{\text{gap}}$  of the hull (0.1 from inside).

Its inner and outer surface emissivities are  $\epsilon_{\text{gap,in}}$  and  $\epsilon_{\text{gap,out}}$  (0.9), the convective heat transfer coefficient is  $\alpha_{\text{gap}}$  ( $5\frac{\text{W}}{\text{Km}^2}$ ), and the conductivity is  $\lambda_{\text{gap}}$  ( $0.01\frac{\text{W}}{\text{Km}}$ ).

Even the smallest habitats do not require additional heating, as thin layers of vacuum and mirrors can provide any desired insulation.

## 2.4 Coolant

The habitat converts both electrical and light energy completely into heat through devices, life, and light absorption. This leads to warming and humidification of air. Wherever a surface temperature drops below the dew point, condensation occurs, accompanied by effective heat transfer. The surface can cool by radiation or by heat conduction to either a coolant or to a surface exposed to space. If heat transfer through the hull is not enough, a coolant is needed that circulates between the habitat and the radiator. Heat can be carried as sensible heat of a liquid (CoolantType=*Liquid*, default), as latent heat of a vapor (*Vapor*), or the coolant is the habitat air itself (*Air*), combining sensible and latent heat as air humidity condenses in the radiator.

Density  $\rho_{\text{C,L}}$  and heat capacity  $c_{\text{C,L}}$  of the liquid coolant as well as the vapor latent heat  $\phi$  default to the water values of  $10^3\frac{\text{kg}}{\text{m}^3}$ ,  $4280\frac{\text{J}}{\text{kgK}}$ , and  $2.453\frac{\text{MJ}}{\text{kg}}$ , respectively.

## 2.5 Heat Absorption

The habitat temperature varies between  $T_{\text{H,min}}$ , where coolant from the radiator enters the habitat, and  $T_{\text{H,max}}$ , where it leaves (defaults are 12 and 27°C). Liquid and vapor require cooling pipes, which allow setting a desired temperature distribution by varying the heat exchange areas. For air flow, however, temperature and power distributions are coupled - most of the power has to be used at high temperatures, where the rise of water vapor with temperature is larger. This would coincide well with tropical climate agriculture and wildlife.

During pipe passage, the coolant warms by  $\delta T_{\text{abs}}$  (default 20K). The minimum difference between habitat and coolant temperature is given by  $\delta T_{\text{HC}}$  (default 5K). The heat transfer coefficient  $\alpha_{\text{abs}}$  between air and coolant of  $20\frac{\text{W}}{\text{Km}^2}$  assumes an average of dry and wet heat exchange. It determines the necessary exchange area with a surface density of  $\sigma_{\text{abs}}$  ( $3\frac{\text{kg}}{\text{m}^2}$ ). The maximum fraction of the habitat volume that is occupied by the pipes is  $f_{\text{C,max}}$  (0.2).

These parameters are ignored if the coolant is the habitat air, as no additional surfaces are needed. Instead, the relative humidity  $h_{r,\text{out}}$  of outgoing air is important, which defaults to 80% (since warm and humid agricultural regions are located there). The air pressure can be lower than on sea level by reducing nitrogen content (default  $p_{\text{A}}$  of 0.4bar). Continuous air flow is effectively limited to the windy volume fraction  $f_{\text{A}}$ , which is a quarter if half of the garden cross section is free from trees. The inner surface per habitat power  $\psi$  defaults to  $0.01\frac{\text{m}^2}{\text{W}}$  (with  $100\text{m}^2$  of garden per person and a channel of quadratic cross section,  $400\text{m}^2$  per 40kW).

## 2.6 Heat Emission

The optimum pipe network would depend on size and shape of the habitat; for simplicity the scalable geometry of Fig. 2 is assumed.

Radiator and hull emit with an emissivity  $\epsilon$  (default 0.9) to space, which counter-radiates with an effective temperature of  $T_{\text{sky}}$  (default 3K).

The surface density  $\sigma_{\text{em}}$  defaults to  $5 \frac{\text{kg}}{\text{m}^2}$ . For simplicity, the surface density of connection pipes is assumed to be the same as in emission ( $\sigma_{\text{con}} = \sigma_{\text{em}}$ ).

The radiator's extension from the axis is limited to  $\kappa_{\text{em}}$  times the habitat radius (default 2), as might be required for co-rotation with the habitat. The length is increased (and the quadratic radiator shape abandoned) when this limit is reached.

## 2.7 Friction

Friction is overcome by spending additional electric power on pumps or fans. Their efficiency is included as  $\eta_{\text{F}}$  (default 0.8). As an approximation, the friction factor has a minimum value of  $\lambda_{\text{min}}$  (0.005) at high Reynolds numbers.

As the optimum flow velocities depend on habitat size, an approximate optimization is performed if  $b_{\text{opt}}$  is true. The cooling system mass is minimized by varying the power that is spent on maintaining the flow in the habitat interior (heat absorption), in the axial pipes (heat connection), and in the radiator (heat emission). The corresponding fractions  $\chi_{\text{abs,st}}$  of absorbed power,  $\chi_{\text{con,st}}$  of transported, and  $\chi_{\text{em,st}}$  of emitted power, are start values if optimized (the optimized values weakly depend on them as iterations are avoided). They default to 1% each. The air flow velocity is computed from the available volume, so  $\chi_{\text{abs,st}}$  is not used in this case. Their sum is limited by  $\chi_{\text{max}}$  (default is a half).

# 3 Model Results

Figs. 3, 4, 5, and 6 display powers, surfaces, volumes and masses for the default case (with liquid coolant). Fig. 5 additionally shows vapor and air coolant.

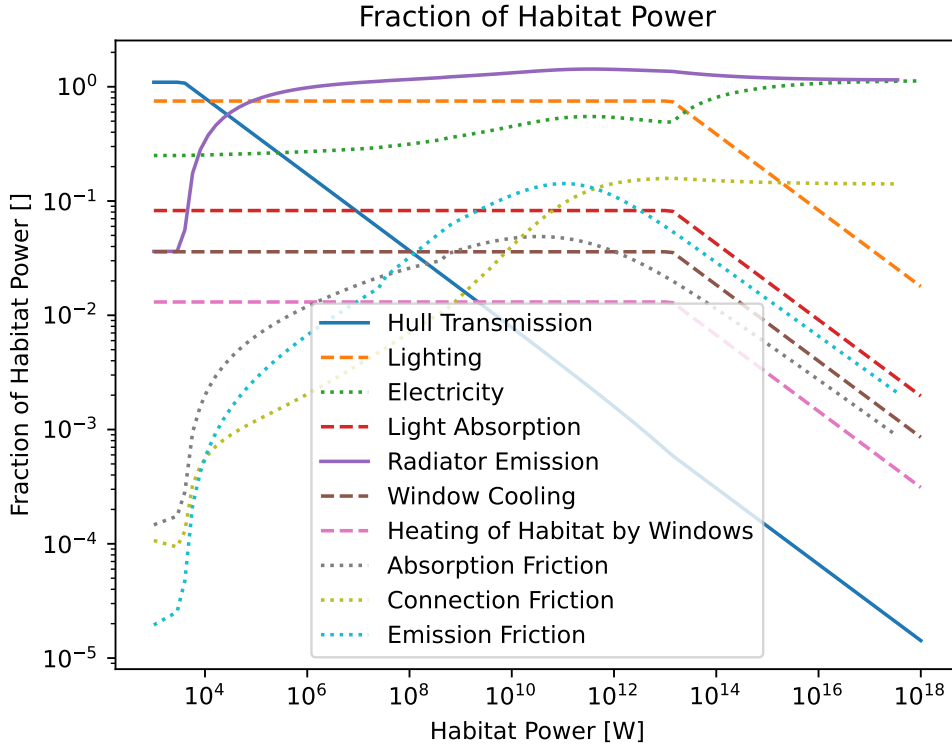
## 3.1 Energy Collection

To meet the lighting demand, an additional 11% is absorbed in light channels and 9% in windows, of which 2% heat the interior and 5% have to be removed by a coolant to prevent overheating (Fig. 3, the remaining 2% are radiated to space).

If every person consumes 10kW of electricity and 30kW of light, PV of about 40m<sup>2</sup> per person is needed, and the cross section (projected area) of primary mirrors is 75m<sup>2</sup>. Due to the concentration, the window area is only 0.2m<sup>2</sup>. Natural illumination (unconcentrated lighting) is possible up to 500kW, where the projected mirror area surpasses the habitat cross section. The radius of the energy collection system is the square root of power times 0.03m (e.g. 30m at 10<sup>6</sup>W and 10km at 10<sup>11</sup>W), of which mirrors take up the inner 80% of the radius.

The volume of light channels is proportional to power and to habitat radius (which itself grows as the third root of power). It takes up 4% of the habitat interior at 10<sup>11</sup>W. The maximum light channel volume fraction is reached at 1.5 10<sup>13</sup>W, where electricity has to supplement the lighting.

The energy collection system is of relatively low mass ( $5 \frac{\text{g}}{\text{W}}$  for PV and  $3 \frac{\text{g}}{\text{W}}$  for mirrors). It is the only component that depends on the distance to the Sun - its area grows quadratically with distance, as the solar power decreases. However, very thin mirrors would keep the mass low as light can also be concentrated on PV, so a lower surface density can partly compensate the larger surface.



**Figure 3:** Powers in the default case as fractions of the habitat power, which is the habitat’s demand of lighting and electricity. Provision of sunlight (denoted as “Lighting”) comes with absorption in light channels (“Light Absorption”) and in the windows, which is partly radiated to space (not shown), heats the habitat (“Heating of Habitat by Windows”), and has to be actively cooled (“Window Cooling”). Part of the habitat power is transmitted through the hull (“Hull Transmission”), but the rest is also actively cooled by carrying a coolant to the radiator (“Radiator Emission”). The latter also includes the friction in the cooling system itself (“Absorption Friction” in the interior, “Connection Friction” in the connection pipe, and “Emission Friction” in the radiator). Those powers are optimized to minimize the total mass. Electricity generation (“Electricity”) covers the habitat demand, the friction power, and the replacement of sunlight above  $10^{13}$ W.

### 3.2 Hull Transfer

The default conditions lead to a heat transfer through the hull of  $26 \frac{\text{W}}{\text{m}^2}$ , and the habitat is naturally cooled only up to a power of 4kW. The fraction that is naturally dissipated falls below 10% at 4MW.

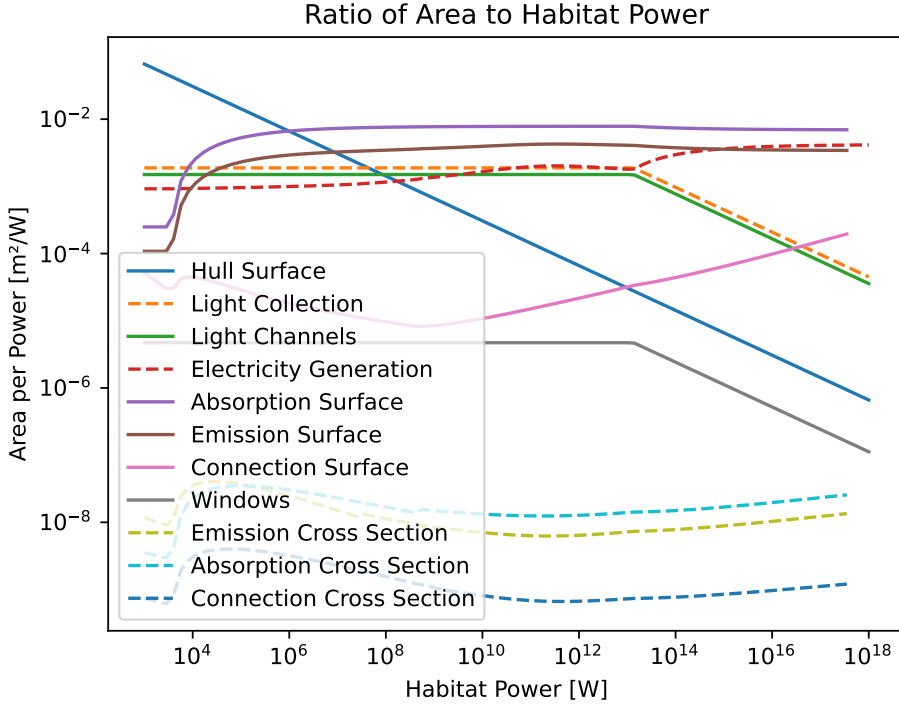
A water/antifreeze-filled hull would combine shielding and good thermal coupling by convection. With an effective heat conductivity of  $1000 \frac{\text{W}}{\text{Km}}$  and no gap ( $d_{\text{gap}} = 0$ ),  $270 \frac{\text{W}}{\text{m}^2}$  is reached, and natural cooling up to 7MW is possible.

The hull has more volume than the habitat below  $6 \cdot 10^5$ W, and more mass below  $2 \cdot 10^9$ W. Its mass falls below  $1 \frac{\text{kg}}{\text{m}^2}$  at  $4 \cdot 10^{10}$ W.

### 3.3 Liquid Cooling

Optimized friction fractions grow from about  $10^{-3}$  at  $10^3$ W to about  $10^{-1}$  at  $10^{11}$ W. At even higher sizes, connection coolant mass dominates, and the friction fractions in heat absorption and emission decrease with size to lower the total cooling power, while the





**Figure 4:** Ratio of areas to habitat power in the default case. For the hull surface, this ratio decreases with larger size as the habitat power is proportional to the interior volume. The cross section of primary mirrors (denoted "Light Collection") is proportional to the habitat power (as are light channel and window surfaces) until lighting by sunlight becomes unfeasible at  $10^{13}$ W. The cooling pipe surfaces and cross sections are almost proportional to habitat power when hull transmission is negligible (above  $10^5$ W).

connection friction stays the same. This is because the connection length grows with size, so more friction power means not only a faster flow, but also a longer connection pipe as all that power has to be emitted (in addition to more PV and emission areas). This limits the usefulness of further increasing  $\chi_{\text{con}}$ .

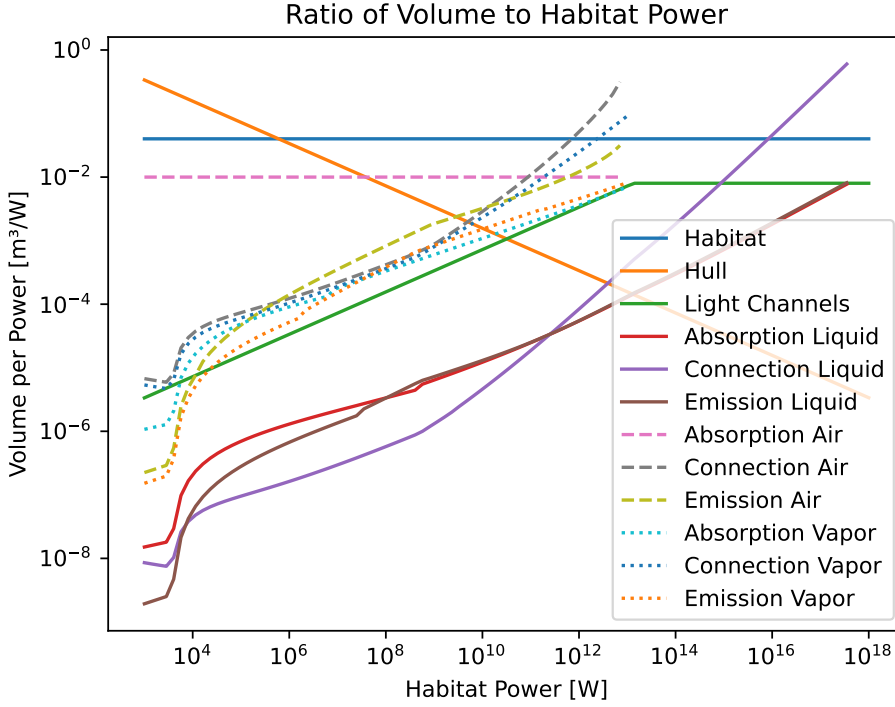
The radiator's effective temperature is 285K. Above  $10^9$ W, its radial extension is limited (by twice the habitat radius).

The fraction of the habitat volume that is occupied by the coolant is generally low (below  $10^{-3}$  at  $10^{11}$ W). It reaches the given maximum above  $10^{17}$  W.

The total cooling mass is dominated by surface masses below  $10^{11}$ W, where it is nearly proportional to the cooling power with about  $0.05 \frac{\text{kg}}{\text{W}}$  (2% of the assumed interior mass). At higher powers the coolant mass dominates, which grows stronger with power due to the increased distance between absorption and emission (amounting to  $0.12 \frac{\text{kg}}{\text{W}}$  at  $10^{11}$ W). It dominates the total mass above  $10^{14}$ W.

The cooling mass flow per power is  $10^{-5} \frac{\text{kg}}{\text{sW}}$ . The mass flow and thus the total coolant mass are inversely proportional to the temperature range  $\delta T_{\text{abs}}$  (between incoming and outgoing coolant). While a lower incoming temperature would mean less absorption surface, it increases the emission surface and hence the coolant mass in emission and connection (and it is limited by avoidance of freezing).

Optimizing the flow velocities can halve the total cooling mass (e.g from 5 to  $2.5 \frac{\text{kg}}{\text{W}}$  at  $10^{14}$ W). This is mainly due to the faster connection flow at high powers.



**Figure 5:** Ratio of volumes to habitat power for the three cooling methods. The hull volume is given by its surface times the shielding thickness. With growing habitat size, light channels take up a larger fraction of the interior habitat volume, which limits the direct use of sunlight. Similarly, the coolant volume would occupy too much of the habitat volume above  $10^{17}$ W in the case of liquid coolant and above  $4 \cdot 10^{12}$ W for vapor cooling. For air, the required friction would be too high above  $10^{13}$ W.

### 3.4 Vapor Cooling

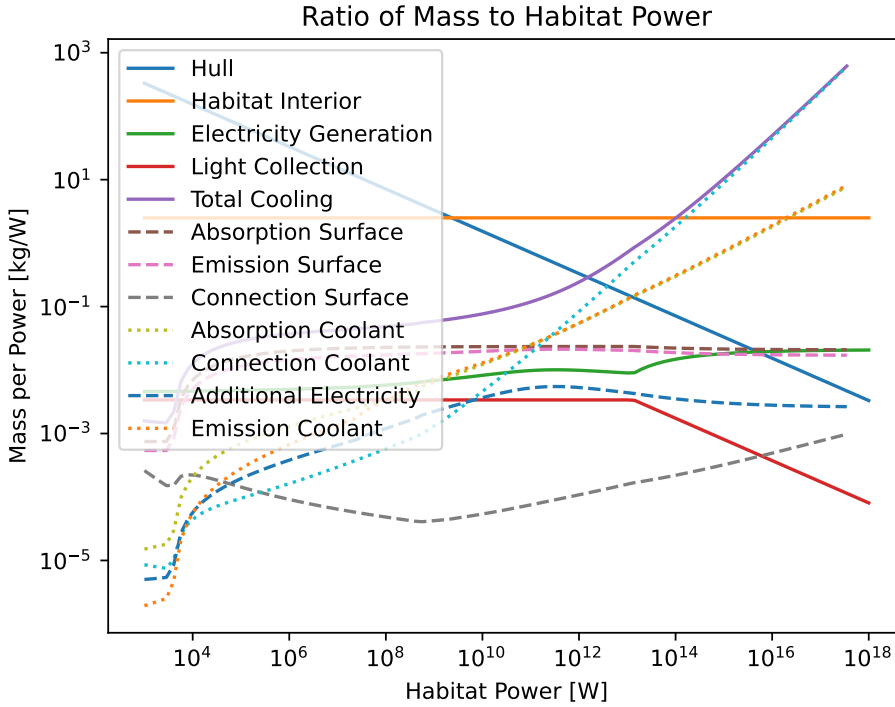
Due to the low density, 5% of the habitat volume is occupied by the coolant at  $10^{11}$ W, and the maximum volume fraction is reached at  $10^{13}$ W. With a faster interior flow ( $\chi_{\text{abs}} = 0.05$ ) or twice the volume per power, cooling remains possible up to  $4 \cdot 10^{13}$ W. Note that the implemented minimization of mass ignores the volume fraction.

The latent heat of water is around 30 times higher than the heat capacity of liquid water with 20K temperature range, reducing the mass flow correspondingly. Also, the low density allows higher flow velocities at the same friction power, decreasing the coolant mass in outgoing pipes by another factor of 50 (incoming pipes are liquid, though). Due to these two effects, the coolant mass is almost two orders of magnitude less than for liquid cooling. Total cooling mass is then usually dominated by the surfaces of absorption and emission.

### 3.5 Air Cooling

A limit of air cooling is reached near  $10^{13}$ W, where wind speeds are so high ( $42 \frac{\text{m}}{\text{s}}$ ) that the given maximum friction power is reached. An alternative to higher wind speeds is larger floor heights - doubling both volume and maximum fraction pushes the limit to  $4 \cdot 10^{14}$ W.

Air flow of default temperatures and humidity carries 70% latent and 30% sensible heat (the latent fraction is reduced to 50% at 1 bar air pressure). Compared to liquid



**Figure 6:** Ratio of masses to habitat power in the default case. Hull (shielding) mass per power decreases with the surface-to-volume ratio. By design, volume and hence the interior mass of the habitat are proportional to the habitat power. Electricity generation mass per power slightly varies due to optimized coolant flow velocities, while the light collection mass is proportional to power until sunlight is replaced by electric lighting. The total cooling mass is the sum of the surface and coolant masses and the additional electricity for keeping the coolant in motion. It grows with power due to the longer distances between heat absorption and emission.

water, the mass flow is 30% higher, but friction is decreased due to the low density. Higher air temperature means more humidity and hence a lower mass flow.

As habitat ground and air double-serve as absorption surface and coolant, no additional heat absorption mass is needed. The total coolant mass at  $10^{11}\text{W}$  is  $0.03 \frac{\text{kg}}{\text{W}}$  (wind speed  $7 \frac{\text{m}}{\text{s}}$ ).

## 4 Discussion and Conclusions

Matter is precious in space, so the mass of a space habitat can be a proxy for cost (which relates to the human work involved and is much harder to estimate). Although this is certainly not proportional, a correlation to launch, transportation, and processing costs can be expected. Matter is more efficiently used the higher the fraction of interior habitat mass. The remaining mass is needed as a service to this payload.

Easing shielding mass demands a compact shape and good use of the volume. With growing size, more volume per hull area is available, but correspondingly more power per hull area has to be transported (light and electricity inwards, waste heat outwards). The physical model quantifies how this energy flow might work.

A transparent hull that allows both natural cooling (only hull transmission) and direct illumination seems a viable option for small habitats up to a MW. However, an up-scaling is prevented by the low power-to-surface ratio, so the power per volume would have to

decrease with larger size, or only the regions near the hull are cooled and illuminated. An active cooling system is needed for the part of the habitat power that is not conducted through the hull, and mirror systems are needed to direct more sunlight into the interior.

A scalable architecture consists of a (rotating) cylinder with its axis pointing to the Sun and multiple floors. It is surrounded by paraboloid mirrors that allow high concentration, but require co-rotation to stay focused on a window. Alternatively, cylindrically symmetric mirrors focus on a window ring. Due to the solid angle of the Sun, window sizes must be at least a percent of the distance to a mirror (at 1AU), and angular deviation of light grows with concentration. In the interior, a network of light channels can be tapped on demand.

Cooling radiators stretch along the axis, emitting from both sides. A coolant circulates in a pipe network. If the habitat air itself is the coolant, air humidity condenses in the radiators, otherwise on the cooling pipes. Compared to a pure liquid, phase changes decrease the required coolant mass, but increase the pipe volume in the gas phase. Depletion by condensation could even drive the circulation, but in general fans or pumps are needed to overcome friction. The faster the coolant flow, the lower is the coolant mass, but the more electric power has to be spent and additionally dissipated. Since distances between heat absorption and emission grow with size (especially as the radiator width is limited due to co-rotation), cooling becomes increasingly difficult for larger habitats.

For electricity generation, on the other hand, up-scaling would not pose a problem: PV panels can be placed farther away, decoupled from rotation by sliding contacts, and cables allow much higher power per cross section.

An economic size would thus be large enough that the hull has less mass than the interior, but small enough that cooling and lighting are feasible. Also structural support, which is not considered here, favors smaller habitats. Reducing shielding surface density or habitat volume is more effective the higher the mass fraction of shielding, so the smaller the habitat is. On the other hand, a large habitat can be more generous with volume. At shielding surface density of  $5 \frac{t}{m^2}$  and volume per power of  $25 \frac{W}{m^3}$ , shielding falls below  $1 \frac{kg}{W}$  at about  $10^{11}W$ . This corresponds to a cylinder of about 1km radius, home to 2.5 million people at 40kW per person.

Even larger habitats could further reduce mass, but cooling and lighting become increasingly challenging. At lower powers, the systems for energy collection and heat dissipation form only a minor contribution to the mass budget.

Circulation of the habitat air has the lowest mass at habitat powers below  $10^{12}W$ , since habitat ground and air double-serve as absorption surface and coolant. Above that power, the coolant mass in heat connection dominates, and the vapor method has lower mass.

At around  $10^{13}W$ , however, the required wind speeds would be too high and the vapor pipes would take up too much interior volume. Liquid cooling remains possible, but climbs above  $1 \frac{kg}{W}$ .

# Appendices

## A Energy Collection

### A.1 Irradiation

Solar irradiation amounts to  $I = (1 - f_{\text{sh}}) \frac{L_{\odot}}{4\pi d_{\odot}^2}$  (with the shaded fraction  $f_{\text{sh}}$ , solar luminosity  $L_{\odot}$ , and distance from the Sun  $d_{\odot}$ ), which (if unshaded) is about  $1360 \frac{\text{W}}{\text{m}^2}$  at Earth's distance ( $d_{\odot} = 1\text{AU}$ ), around 500 of which is PAR (photosynthetically active radiation, mostly coinciding with visible light).

This constant irradiation could be temporarily blocked by obstacles, such as planetary bodies. At an altitude  $h$  above a planet of radius  $R$ , the planet appears under a radial angle of  $\theta = \arcsin \frac{R}{R+h}$ . In an ecliptic orbit, the Sun is shaded during a fraction  $f_{\text{sh}} = \frac{\theta}{180^\circ}$ . With increasing angular deviation from the ecliptic, this fraction shrinks. The planet covers a fraction of the sky (field of view) that is given by  $f_{\text{sky}} = \sin^2(\theta/2) = \frac{1}{2}(1 - \cos \theta)$ . A sphere at this orbit receives  $f_{\text{sky}}$  times the planetary emission per surface area.

In LEO (low Earth orbit) at 400km altitude,  $\theta$  is  $70^\circ$ , so  $f_{\text{sh}}$  is about 0.4 for the ecliptic, and a third of the sky is covered by Earth. Earth reflects 30% of incoming sunlight ( $100 \frac{\text{W}}{\text{m}^2}$  of surface), adding a diffuse component to near-Earth sunlight. A sphere in LEO receives a third of this, i.e.  $33 \frac{\text{W}}{\text{m}^2}$  of surface area, while unshaded sunlight amounts to  $340 \frac{\text{W}}{\text{m}^2}$  (a quarter of 1360 as its surface is 4 times the projected area). So on average, in LEO 40% of the sunlight is blocked by Earth and 10% is added by reflection as a diffuse component, which makes a net 30% reduction, or  $240 \frac{\text{W}}{\text{m}^2}$  of surface area. The diffuse sunlight cannot be used by energy collection systems that focus on the Sun, but e.g. for natural illumination by a transparent hull.

### A.2 Electricity

The demand for electricity is  $P_{\text{E}} = f_{\text{E}} P_{\text{H}} + P_{\text{F}}$  with the electric fraction of the habitat power  $f_{\text{E}}$  and the friction power in the cooling system  $P_{\text{F}}$  (losses in the electric distribution are small and assumed to be included in the habitat power).

It is produced by photovoltaics (PV) or solar thermal power plants. The necessary electrical (projected) collecting area  $A_{\text{E}}$  is found by  $P_{\text{E}} = \eta_{\text{E}} I A_{\text{E}}$  with the efficiency of converting sunlight to electricity  $\eta_{\text{E}}$ , so  $A_{\text{E}} = \frac{P_{\text{E}}}{\eta_{\text{E}} I}$ .

The electricity generation does not need to co-rotate with the habitat as it can be connected via sliding contacts. The mass of the electrical collector is  $M_{\text{E}} = A_{\text{E}} \sigma_{\text{E}}$  with the surface density of PV modules (incl. wires for distribution)  $\sigma_{\text{E}}$ .

### A.3 Lighting

Sunlight can be collected by a system of mirrors arranged around the habitat. The primary mirrors focus light on windows in the hull (Sec. A.3.1), which transmit it to the interior (Sec. A.3.2). Light channels can then distribute it (Sec. A.3.3).

#### A.3.1 Concentration

Infrared and ultraviolet light is preferably transmitted or absorbed by the primary mirrors (or reflected by the windows), as plants cannot use it for photosynthesis.

With the habitat axis pointing to the Sun, the mirrors provide constant light without moving relative to the habitat, and concentrate in parallel and/or perpendicular directions to the axis. The latter requires co-rotation of paraboloid mirror segments, which implicates high gravity at large radii. Non-rotating mirrors, on the other hand, are stationary and weightless, but can concentrate only in cylindrical symmetry.

The intensity per solid angle cannot be larger than the solar surface brightness times mirror reflectivity, which limits concentration. The concentration factor  $\gamma$  is here defined as the ratio of projected collecting area (of outer mirrors) to window area ( $\gamma = \frac{A_{M,\text{out}}}{A_W}$ ). As seen from a window, the mirror solid angle is  $\gamma$  times the Sun's solid angle. The angle from the Sun's center is the ratio of solar radius to distance  $\frac{R_\odot}{d_\odot}$ , about a quarter degree at 1AU. Assuming concentration in all directions, the maximum angular deviation (from normal) is  $\delta_{\text{max}} = \sqrt{\gamma} \frac{R_\odot}{d_\odot}$  (in radians).

In cylindrical symmetry, the concentration factor is the ratio of the mirror radius to the radius of a window ring times concentration in axial direction. Thus, the angular deviation grows linearly with the axial concentration in this case, and the typical angular deviation is larger. High concentration in both dimensions can be reached by focusing on the axis or a window ring at the habitat's rear side.

The distance between window and mirror  $d_M$  determines the minimum size of the focal spot, i.e. the window radius  $R_W = d_M \frac{R_\odot}{d_\odot}$  (about half a percent of  $d_M$  at 1AU). The outermost mirrors at  $d_{M,\text{max}} = \sqrt{\frac{A_{M,\text{out}}}{\pi}} - R_H$  (with habitat radius  $R_H$ ) illuminate the largest windows. The projected mirror radius is hence  $R_{M,\text{max}} = \sqrt{\gamma} R_{W,\text{max}} = \sqrt{\gamma} d_{M,\text{max}} \frac{R_\odot}{d_\odot} = \delta_{\text{max}} d_{M,\text{max}}$  (note that mirror cross sections could be hexagonal instead of circular to avoid gaps). The next ring of smaller mirrors would start from  $d_{M,\text{max}} - 2R_{M,\text{max}} = (1 - 2\delta_{\text{max}}) d_{M,\text{max}}$  inwards.

### A.3.2 Transmittance

Light enters the pressurized habitat volume through windows, which absorb a fraction  $a_W$  of the incident light and reflect a fraction  $r_W$ . The absorbed power  $P_{W,\text{abs}} = a_W P_W$  is partly emitted to space as  $P_{W,\text{em}} = A_W \epsilon \sigma (T_W^4 - T_{\text{sky}}^4)$  with emissivity  $\epsilon$ , Stefan-Boltzmann constant  $\sigma = 5.67 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$ , window temperature  $T_W$ , and the effective temperature of counter-radiation  $T_{\text{sky}}$  (Sec. B.6). Another part heats the habitat,  $P_{W,H} = A_W \epsilon \sigma (T_W^4 - T_H^4)$  with (average) habitat temperature  $T_H$ . To simplify and to account for insulation by light channels, the convective transfer is neglected here.

Finally, the coolant could pass the windows when leaving the habitat to maintain a maximum window temperature  $T_{W,\text{max}}$ , requiring a cooling power of  $P_{W,C}$ . If the light channel interior would be vacuum, the intensity on interior windows would be less, but that would require more pressure-containing surface and the windows could not cool radiatively.

The window temperature follows from heat balance  $P_{W,\text{abs}} = P_{W,\text{em}} + P_{W,H} + P_{W,C} = A_W \epsilon \sigma (2T_W^4 - T_{\text{sky}}^4 - T_H^4) + P_{W,C}$  to

$$T_W = \sqrt[4]{\frac{1}{2} \left( \frac{P_{W,\text{abs}} - P_{W,C}}{A_W \epsilon \sigma} + T_{\text{sky}}^4 + T_H^4 \right)}$$

If the window temperature without cooling ( $P_{W,C} = 0$ ) is above  $T_{W,\text{max}}$ , cooling is required with  $P_{W,C} = P_{W,\text{abs}} - A_W \epsilon \sigma (2T_{W,\text{max}}^4 - T_{\text{sky}}^4 - T_H^4)$ .

### A.3.3 Distribution

Entering light may be distributed from windows to ceilings by mirror-walled channels (Fig. 1). By moving mirrors inside the channels (or by opening gaps), the constant light power is locally tapped in varying degree, allowing weather, diurnal and seasonal cycles.

The light channels form a meandering network, e.g. consisting of vertical columns with window bases and horizontal branches along the ceilings. Their cross section diminishes as light is tapped. It is beyond the scope of this paper to model this shape, so the channel surface is parameterized as  $A_{\parallel,L} = \frac{(1-f_E)P_H}{I_{\parallel}}$  where  $I_{\parallel}$  is the average intensity of the outer channel surface, shining into the habitat. This intensity should be low enough to allow a wide distribution and to avoid dangerous blending.

Absorption on the channel walls is the surface times the mirror absorptivity  $(1 - r_{M,in})$  times the projected intensity. The latter is obtained by integrating the brightness over the solid angle. Seen from a mirror, a half-circle of approximately uniform brightness at the edge of the field of view allows integration over the angular deviation. Neglecting that brightness diminishes with distance from the window (more the higher the angular deviation), the absorbed power in light distribution is  $P_{D,abs} = A_{\parallel,L}(1 - r_{M,in})B_{\odot}(1 - f_{sh})r_{M,out}(1 - r_W - a_W) \int_0^{\delta_{max}} \pi \delta \sin \delta d\delta$  where  $B_{\odot} = \frac{L_{\odot}}{4\pi^2 R_{\odot}^2}$  is the solar brightness (intensity of  $\frac{L_{\odot}}{4\pi d_{\odot}^2}$  per solid angle of  $\frac{\pi R_{\odot}^2}{d_{\odot}^2}$ ). Assuming small angular deviation ( $\sin \delta \approx \delta$ ) leads to

$$P_{D,abs} = A_{\parallel,L}(1 - r_{M,in}) \frac{(1 - f_{sh})L_{\odot}}{12\pi R_{\odot}^2} r_{M,out}(1 - r_W - a_W) \delta_{max}^3$$

For cylindrically symmetric mirrors, the window ring can be placed at lower radius and be partitioned into separate light channels with similar angular deviation in both directions.

In a cylindrical habitat of radius  $R_H$  and uniform power distribution, the average distance to the hull can be found by integrating over the radius to be  $\frac{\int_0^{R_H} (R_H - r) 2\pi r dr}{\int_0^{R_H} 2\pi r dr} = \frac{1}{3}R_H$ .

The channel volume follows from the cross section that is needed to transmit a power of  $(1 - f_E)P_H + P_{D,abs}$  with intensity (power per cross section)  $I_L = r_{M,out}(1 - r_W - a_W)\gamma I$ .

$$V_L = \frac{(1 - f_E)P_H + P_{D,abs}}{3I_L} R_H$$

The habitat radius is derived from the habitat volume  $V_H = \frac{P_H}{\nu_P}$  (with power density  $\nu_P$ ) and the aspect ratio  $\xi$  as  $R_H = \sqrt[3]{\frac{V_H}{\pi\xi}}$ .

Irradiation on windows is required to be  $P_W = (1 - f_E)P_H + P_{D,abs} + P_{W,abs} + P_{W,refl}$ , so  $P_W = \frac{(1-f_E)P_H + P_{D,abs}}{1 - a_W - r_W}$ . The cross section of primary mirrors follows to be  $A_{M,out} = \frac{P_W}{r_{M,out}I}$ , and the mirror mass is  $M_L = \sigma_L(A_{M,out} + A_{\parallel,L})$  (where  $\sigma_L$  is the average surface density of the light collection system).

Natural illumination by direct sunlight without mirrors is possible if the collecting area  $A_{M,out}$  is smaller than the habitat cross section. If the channel volume  $V_L$  surpasses a given maximum fraction  $f_{L,max}$  of the habitat volume  $V_H$  (or the window area  $A_W$  approaches the hull surface), sunlight is replaced by electric lighting (higher  $f_E$ ).

## B Heat Dissipation

In addition to passive heat propagation through the hull (Sec. B.1), three methods of active heat transportation can be computed by the model: liquid, vapor, and air (Sec. B.2).

Also, three components of the cooling system are treated separately: heat absorption in the habitat (Secs. B.3 and B.4), the connection to the radiator (Sec. B.5), and heat emission by the radiator (Sec. B.6). These are denoted by subscripts  $_{\text{abs}}$ ,  $_{\text{con}}$ , and  $_{\text{em}}$ , respectively. Sec. B.7 describes the friction computation, Sec. B.8 a mass minimization by varying the friction power.

A sketch of the system is shown in Fig. 2.

### B.1 Heat Propagation through the Hull

Heat dissipates through the hull in addition to - or instead of - a cooling system. In this section, the transmission power is derived for a hull that consists of an outer and inner part with a gap between them. Such gaps are necessary if the shielding does not rotate with the habitat, or if the pressure containment has two layers. The computation is iterated to find the transmitted power per surface  $\Psi$  for a given hull.

Starting from the interior with air temperature  $T_{\text{H,min}}$  close to the hull, the inner hull temperature is  $T_{\text{hull,in}} = T_{\text{H,min}} - \frac{\Psi}{\alpha}$ . The heat transfer coefficient between the inner hull surface and the habitat air  $\alpha$  is the sum of the coefficients due to convection (depending on air movement), radiation (around  $5 \frac{\text{W}}{\text{m}^2\text{K}}$ ), and condensation (depending on humidity). Condensation results in a large  $\alpha$ , so the inner hull temperature would approach at least the dew point of the habitat air.

The resistance of the inner hull is  $\mathfrak{R}_{\text{in}} = f_{\text{in}} \frac{\sigma_{\text{S}}}{\rho_{\text{S}} \lambda_{\text{S}}}$ , where  $f_{\text{in}}$  is the fraction of the hull inside the gap,  $\sigma_{\text{S}}$  the shielding surface density,  $\rho_{\text{S}}$  shielding density, and  $\lambda_{\text{S}}$  shielding conductivity. The inner temperature of the gap follows to be  $T_{\text{gap,in}} = T_{\text{hull,in}} - \Psi \mathfrak{R}_{\text{in}}$ .

Heat is propagated through the gap by exchange of thermal radiation as well as conduction and convection. On each side, the incoming radiation consists of the emission from the other side, the reflected own emission, the doubly reflected other emission, the triply reflected own emission, and so on. For given surface temperatures the net transmitted radiative power per surface  $\Psi_{\text{rad}}$  can be found by summing these contributions.

The original emissions are  $\epsilon_{\text{gap,in}} \sigma T_{\text{gap,in}}^4$  and  $-\epsilon_{\text{gap,out}} \sigma T_{\text{gap,out}}^4$ . In the first step, they are multiplied by the negative reflectivities  $-r_{\text{out}}$  and  $-r_{\text{in}}$ , respectively, then vice versa. These steps are repeated until the change in  $\Psi_{\text{rad}}$  falls below a threshold (e.g. 1%). The reflectivities are  $r_{\text{in}} = 1 - \epsilon_{\text{gap,in}}$  and  $r_{\text{out}} = 1 - \epsilon_{\text{gap,out}}$ , since the absorptivity to its own radiation equals the emissivity, and no transparency is assumed. Also, the two surface temperatures are assumed to be sufficiently similar that both emissions are equally reflected.

If the gap is filled with gas, the gas additionally transports heat outwards by conduction and convection. The convective resistance is  $\mathfrak{R}_{\text{conv}} = \frac{2}{\alpha_{\text{gap}}}$  with the convective heat transfer coefficient in the gap  $\alpha_{\text{gap}}$  (on both sides), the conductive resistance is  $\mathfrak{R}_{\text{cond}} = \frac{d_{\text{gap}}}{\lambda_{\text{gas}}}$  with the heat conductivity of the gas  $\lambda_{\text{gas}}$ . Both convection ( $\alpha_{\text{gap}}$ ) and conduction ( $\lambda_{\text{gas}}$ ) decrease with lower gas density. The radiative resistance is  $\mathfrak{R}_{\text{rad}} = \frac{T_{\text{gap,in}} - T_{\text{gap,out}}}{\Psi_{\text{rad}}}$ . This quantity is only weakly dependent on the choice of  $T_{\text{gap,out}}$  (1K difference is assumed in the implementation). As radiative, convective and conductive resistances are parallel, the resulting gap resistance is  $\mathfrak{R}_{\text{gap}} = (\mathfrak{R}_{\text{conv}}^{-1} + \mathfrak{R}_{\text{cond}}^{-1} + \mathfrak{R}_{\text{rad}}^{-1})^{-1}$ , and the outer gap temperature can be finally computed as  $T_{\text{gap,out}} = T_{\text{gap,in}} - \Psi \mathfrak{R}_{\text{gap}}$ .



The resistance of the outer hull is  $\mathfrak{R}_{\text{out}} = (1 - f_{\text{in}}) \frac{\sigma_{\text{S}}}{\rho_{\text{S}} \lambda_{\text{S}}}$ , so the surface temperature is  $T_{\text{S}} = T_{\text{gap,out}} - \Psi \mathfrak{R}_{\text{out}}$ . The hull emits with this temperature, but receives thermal radiation from surrounding objects (absorptivity of which is assumed to be the same as the emissivity  $\epsilon$ ), and a fraction  $\frac{\pi R_{\text{H}}^2}{A_{\text{Hull}}} = \frac{\pi R_{\text{H}}^2}{2\pi R_{\text{H}}^2 + 2\pi R_{\text{H}} \xi R_{\text{H}}} = \frac{1}{2+2\xi}$  of the hull absorbs sunlight with the absorptivity  $a_{\text{S}}$ . This leads to a net emitted power of  $\Psi_{\text{em}} = \epsilon \sigma (T_{\text{S}}^4 - T_{\text{sky}}^4) - a_{\text{S}} I \frac{1}{2+2\xi}$ . The computation is repeated with a modified  $\Psi$  until  $\Psi \approx \Psi_{\text{em}}$ .

The power that is dissipated inside the shielding is  $P_{\text{in}} = f_{\text{P,in}} P_{\text{H}} + P_{\text{D,abs}} + P_{\text{W,H}}$ , where  $f_{\text{P,in}}$  is the fraction of habitat power inside the shielding. Windows and light channels additionally heat the habitat. As the hull can easily have additional insulation by vacuum layers, the hull power is  $P_{\text{Hull}} = \min(P_{\text{in}}, \Psi(A_{\text{Hull}} - A_{\text{W}}))$ .

The required cooling power can be found to be  $P_{\text{C}} = P_{\text{in}} - P_{\text{Hull}} + P_{\text{W,C}}$ . The cooling system to provide that power is modeled in the rest of the appendix.

## B.2 Coolant Flow

The power per mass flow that is exchanged within a certain region is the change in internal energy per coolant mass  $\delta u$ ,

$$P = \dot{m} \delta u = \dot{m} (c_{\text{C}} \Delta T + \phi \Delta h)$$

with the (average) heat capacity  $c_{\text{C}}$  ( $\approx 4.2 \frac{\text{J}}{\text{gK}}$  for liquid water, 1 for air, and 1.9 for water vapor), the latent heat  $\phi$  ( $\approx 2453 \frac{\text{J}}{\text{g}}$  for water) and the specific humidity (water vapor fraction of mass flow)  $h = \frac{\dot{m}_{\text{w,g}}}{\dot{m}}$ .

For liquid  $h = 0$ , so  $\delta u = c_{\text{C}} \Delta T$ , while for (completely condensing) vapor the latent heat dominates,  $\delta u = c_{\text{C}} \Delta T + \phi \approx \phi$ .

For air, the specific humidity is  $h = \frac{\rho_{\text{w,g}}}{\rho_{\text{C,A}}}$  with the densities of gaseous water  $\rho_{\text{w,g}}$  and air  $\rho_{\text{C,A}}$  (including gaseous and liquid water). Applying the ideal gas law  $p = nk_{\text{B}}T = \frac{\rho}{m} k_{\text{B}}T$  with the particle density  $n$ , the molecular mass  $m$  and the Boltzmann constant  $k_{\text{B}}$  leads to  $\rho \sim mp$  and  $h = \frac{m_{\text{w}} p_{\text{w}}}{m_{\text{A}} p_{\text{A}}}$ . The molecular mass of water is  $m_{\text{w}} = 18\text{u}$  and the average molecular mass of air (half oxygen with 32u and half nitrogen with 28u) is around 30u. As this includes water in both phases, the water vapor content lowers  $m_{\text{A}}$ , while the liquid water fraction effectively increases it (less gas molecules per mass). The air pressure  $p_{\text{A}}$  is a constant input, the water pressure is  $p_{\text{w}} = h_r p_{\text{w,S}} \approx h_r p_{\text{w,tp}} e^{\frac{\phi}{R_{\text{w}}} (\frac{1}{273} - \frac{1}{T})}$  with the relative humidity  $h_r$ , the saturation pressure at triple point  $p_{\text{w,tp}} = 611\text{Pa}$  and the water gas constant  $R_{\text{w}} = \frac{k_{\text{B}}}{m_{\text{w}}} = 461 \frac{\text{J}}{\text{kgK}}$ . This formula, an integrated form of the Clausius-Clapeyron relation, neglects the temperature-dependence of the latent heat, but is accurate enough for this purpose. The change in specific humidity at constant air pressure can thus be approximated as  $\Delta h = \frac{18 p_{\text{w,tp}}}{30 p_{\text{A}}} \left( h_{r1} e^{\frac{\phi}{R_{\text{w}}} (\frac{1}{273} - \frac{1}{T_1})} - h_{r2} e^{\frac{\phi}{R_{\text{w}}} (\frac{1}{273} - \frac{1}{T_2})} \right)$ .

The mass flow is  $\dot{m} = w A_{\perp} \rho_{\text{C}}$ , with the velocity  $w$ , the cross section  $A_{\perp}$  (area perpendicular to the flow), and the density of the coolant  $\rho_{\text{C}}$ . The liquid coolant density is a direct input parameter  $\rho_{\text{C,L}}$  ( $10^3 \frac{\text{kg}}{\text{m}^3}$  for water), while the air density is approximated as  $\rho_{\text{C,A}} = \frac{p_{\text{A}}}{R_{\text{A}} T_{\text{C}}} \approx \frac{p_{\text{A}}}{10^5 \text{Pa}} 1.2 \frac{\text{kg}}{\text{m}^3}$ . The density of an evaporating coolant changes dramatically while undergoing phase transition (requiring large outgoing and small incoming pipes). In the gas phase, it is  $\rho_{\text{C,V}} = \frac{2 p_{\text{w}}}{R_{\text{w}} T_{\text{C}}} = \frac{p_{\text{w,tp}}}{2 R_{\text{w}} T_{\text{C}}} e^{\frac{\phi}{R_{\text{w}}} (\frac{1}{273} - \frac{1}{T_{\text{C}}})}$  with the boiling point  $T_{\text{C}}$ . Coolant volume and mass are approximated as the average of gas and liquid (using their corresponding densities and velocities).

### B.3 Heat Absorption (Liquid and Vapor)

If the coolant temperature increase during heat absorption  $\delta T_{\text{abs}}$  is larger than the habitat temperature spread  $T_{\text{H,max}} - T_{\text{H,min}}$ , the outgoing temperature is  $T_{\text{C,out}} = T_{\text{H,max}} - \delta T_{\text{HC,min}}$  and the incoming temperature follows to be  $T_{\text{C,in}} = T_{\text{C,out}} - \delta T_{\text{abs}}$ . If it is smaller, then  $T_{\text{C,in}} = T_{\text{H,min}} - \delta T_{\text{HC,min}}$  and  $T_{\text{C,out}} = T_{\text{C,in}} + \delta T_{\text{abs}}$ .

The internal energy change of a liquid coolant is  $\delta u_{\text{abs}} = c_{\text{C}}(T_{\text{C,out}} - T_{\text{C,in}})$ , while vapor has  $\delta u_{\text{abs}} = c_{\text{C}}(T_{\text{C,out}} - T_{\text{C,in}}) + \phi$ . To compensate friction  $\dot{m}$  is increased, so it absorbs not only the habitat power but also the friction power in heat absorption, which is parameterized as  $P_{\text{F,abs}} = \chi_{\text{abs}} P_{\text{C}}$ .

$$\dot{m} = \frac{P_{\text{C}} + P_{\text{F,abs}}}{\delta u_{\text{abs}}} = \frac{(1 + \chi_{\text{abs}})P_{\text{C}}}{\delta u_{\text{abs}}}$$

Heat exchange between habitat and coolant is  $\Delta P_{\text{C}} = \alpha (T_{\text{H}} - T_{\text{C}}) \Delta A_{\parallel}$ , where  $\alpha$  is the heat transfer coefficient,  $T_{\text{H}}$  the habitat temperature,  $T_{\text{C}}$  the coolant temperature, and  $A_{\parallel}$  the pipe surface area (parallel to the flow). During heat absorption,  $T_{\text{H}}$  increases from  $T_{\text{H,min}}$  to  $T_{\text{H,max}}$  and  $T_{\text{C}}$  from  $T_{\text{C,in}}$  to  $T_{\text{C,out}}$ . If their difference  $\delta T_{\text{HC}} = T_{\text{H}} - T_{\text{C}}$  is constant (i.e. the habitat warms as much as the coolant), integration over the power gives

$$A_{\parallel,\text{abs}} = \frac{P_{\text{C}}}{\alpha \delta T_{\text{HC}}}$$

If  $\delta T_{\text{HC}}$  is not constant, the integration can be carried out over  $\delta T_{\text{HC}}$  as both  $T_{\text{H}}$  and  $T_{\text{C}}$  change linearly with power (making use of  $\frac{\Delta P_{\text{C}}}{P_{\text{C}}} = \frac{\Delta(\delta T_{\text{HC}})}{\delta T_{\text{HC,max}} - \delta T_{\text{HC,min}}}$ ), so  $A_{\parallel,\text{abs}} = \int_{\delta T_{\text{HC,min}}}^{\delta T_{\text{HC,max}}} \frac{P_{\text{C}}}{\alpha(\delta T_{\text{HC,max}} - \delta T_{\text{HC,min}})\delta T_{\text{HC}}} d(\delta T_{\text{HC}})$  yields

$$A_{\parallel,\text{abs}} = \frac{P_{\text{C}}}{\alpha(\delta T_{\text{HC,max}} - \delta T_{\text{HC,min}})} \ln \frac{\delta T_{\text{HC,max}}}{\delta T_{\text{HC,min}}}$$

The minimum temperature difference  $\delta T_{\text{HC,min}}$  is an input parameter, the maximum is  $\delta T_{\text{HC,max}} = \max(T_{\text{H,max}} - T_{\text{C,out}}, T_{\text{H,min}} - T_{\text{C,in}})$  (either at end or start of the absorption pipes).

For vapor cooling, the coolant temperature in the liquid phase can not surpass the boiling point, and the vapor temperature is not much larger due to sufficient contact with the liquid phase and continual wetting of absorbing surfaces. The coolant temperature can thus be set by varying the pipe pressure (e.g. through variations of the cross section and the fans), which requires additional electric power. For simplicity, the same two absorption surface approximations as for liquid are used. This likely overestimates the surface, as most surfaces are close to the boiling point and thus on average colder than for liquid.

A vapor cooling system with lower pressure in the radiator could actually work without actively driving the flow, i.e. without electrical power, just by the pulling force of condensation in the radiator, where the pressure is kept lower by the continual depletion of vapor molecules. To provide the driving force, the pressure decreases in the course of both heat absorption and emission, and so does the boiling point, while the cross section increases in the habitat and decreases in the radiator. The radiator would have to be located at lower gravity than the habitat, so the liquid can flow downwards. Condensation would occur at lower temperatures, however, requiring a larger emitting area. Conversely, a reverse pressure gradient could be achieved by active compression, leading to higher condensation than evaporation temperatures.

As the coolant is assumed to flow back and forth from the axis with  $R_{\text{abs}} = R_{\text{H}}$ , its volume is  $V_{\text{abs}} = 2R_{\text{abs}}A_{\perp,\text{abs}} = \frac{2R_{\text{abs}}\dot{m}}{\rho_C w_{\text{abs}}}$  (as the cross section is  $A_{\perp,\text{abs}} = \frac{\dot{m}}{\rho_C w_{\text{abs}}}$ ). The relation between velocity and friction power is derived in Sec. B.7.

The passage time (that the coolant spends in heat absorption) is  $\frac{2R_{\text{abs}}}{w_{\text{abs}}}$  and the coolant mass is the mass flow times this passage time. The total mass in heat absorption is the sum of coolant and pipe mass  $M_{\text{abs}} = \frac{2R_{\text{abs}}\dot{m}}{w_{\text{abs}}} + \sigma_{\text{abs}}A_{\parallel,\text{abs}}$  with the surface density  $\sigma_{\text{abs}}$ .

## B.4 Heat Absorption (Air)

Incoming and outgoing temperatures are given by the minimum and maximum habitat temperatures,  $T_{\text{C},\text{in}} = T_{\text{H},\text{min}}$  and  $T_{\text{C},\text{out}} = T_{\text{H},\text{max}}$ .

$$\text{As derived in Sec. B.2, } \delta u_{\text{abs}} = c_{\text{C}}(T_{\text{C},\text{out}} - T_{\text{C},\text{in}}) + \phi \frac{18p_{\text{w},\text{tp}}}{30p_{\text{A}}} \left( h_{r,\text{out}} e^{\frac{\phi}{R_{\text{w}}}\left(\frac{1}{273} - \frac{1}{T_{\text{C},\text{out}}}\right)} - e^{\frac{\phi}{R_{\text{w}}}\left(\frac{1}{273} - \frac{1}{T_{\text{C},\text{in}}}\right)} \right)$$

with the relative humidity of outgoing air  $h_{r,\text{out}}$ . The incoming air is saturated with water vapor by the cooling process ( $h_{r,\text{in}} = 1$ ).

As for liquid and vapor, the mass flow that takes up the habitat's heat ( $\frac{P_{\text{C}}}{\delta u_{\text{abs}}}$ ) is increased to absorb also the heat from friction. Contrary to the other coolants, however, not the friction power per cooling power  $\chi_{\text{abs}}$  is given, but the free air volume as the fraction of the habitat volume  $f_{\text{A}}$  ( $V_{\text{abs}} = f_{\text{A}}V_{\text{H}}$ ). The cross section is hence  $A_{\perp,\text{abs}} = \frac{f_{\text{A}}V_{\text{H}}}{2R_{\text{H}}}$  (assuming a typical length of twice the habitat radius) and the wind speed is  $w_{\text{abs}} = \frac{\dot{m}}{\rho_{\text{C}}A_{\perp,\text{abs}}} = \frac{2R_{\text{H}}\dot{m}}{\rho_{\text{C}}f_{\text{A}}V_{\text{H}}}$ . The friction follows as computed in Sec. B.7, and the mass flow has to be determined iteratively. Note that wind speeds would vary with height and location due to obstacles. Typically, the flow is faster in the upper part of each floor, and near-ground air is a bit warmer than the air flow.

The habitat consists of alternating layers of ground and air. With the inner surface per cooling power  $\psi$ ,

$$A_{\parallel,\text{abs}} = \psi P_{\text{C}}$$

The absorption coolant mass is already counted as the habitat air, and the heat exchange surface is counted as the ground, both of which are included in the interior mass, so  $M_{\text{abs}} = 0$ .

## B.5 Heat Connection

To bring the coolant from the habitat to the radiator and back, the model assumes it is collected at the axis and distributed to absorption and emission pipes.

The length of one half of the central connection pipe is  $L_{\text{abs}} + L_{\text{con}} + L_{\text{em}}$  with the length inside the habitat (cylinder half-length)  $L_{\text{abs}} = \frac{V_{\text{H}}}{2\pi R_{\text{abs}}^2}$ , the radiator length  $L_{\text{em}}$  (Sec. B.6), and the gap between heat absorption and emission (hull thickness)  $L_{\text{con}} = \frac{\sigma_{\text{S}}}{\rho_{\text{S}}}$ . The cross section diminishes linearly with distance from the hull in both absorption and emission, as a portion of the coolant that has absorbed or emitted its share of heat is transported back.

Note that in heat absorption and emission the coolant is assumed to flow the whole way to the edge, although it could also partially return once heat is exchanged. This could save up to half of the coolant mass in heat absorption and emission, at the expense of additional surfaces. Also, connection pipes can be shorter if habitat regions are directly connected to nearby radiator parts - this effect is strongest in small habitats, which have low connection coolant mass anyway. An optimal configuration would probably

be a meandering network of connection pipes, balancing the shorter distances with the increased friction and surface mass.

The connection pipe volume is the integrated cross section. If the cross section is proportional to length, the volume is half of what it would be with constant maximum cross section ( $\int_0^1 x dx = \frac{1}{2}$  with a normalized length  $x$ ). The surface area of the connection pipe is the integrated circumference, which diminishes as the square root of the cross section and hence of length. An integration over the length yields two thirds of the constant solution ( $\int_0^1 x^{\frac{1}{2}} dx = \frac{2}{3}$ ), so the effective length for computing the surface area is  $L_{\text{eff}} = \frac{2}{3}(L_{\text{abs}} + L_{\text{em}}) + L_{\text{con}}$ .

The temperature increase due to friction is  $\Delta T_{\text{con}} = \frac{P_{\text{F,con}}}{2c_C \dot{m}}$  in both directions. This has to be additionally emitted by the radiator, leading to a broader temperature range and an approximately linear increase of its area with the friction power  $P_{\text{F,con}} = \chi_{\text{con}}(P_C + P_{\text{F,abs}}) = \chi_{\text{con}}(1 + \chi_{\text{abs}})P_C$ . In the case of air cooling, the incoming air is still assumed to be saturated with water vapor, although its relative humidity would be lowered by this warming.

The total mass of the connection from habitat to radiator is  $M_{\text{con}} = \frac{2\dot{m}}{w_{\text{con}}} \left( \frac{L_{\text{abs}} + L_{\text{em}}}{2} + L_{\text{con}} \right) + \sigma_{\text{con}} A_{\parallel, \text{con}}$  with the factors of 2 for the two directions and of a half for the diminishing cross section. The habitat volume that is occupied by the connection pipe is  $V_{\text{con,H}} = \frac{\dot{m} L_{\text{abs}}}{\rho_C w_{\text{con}}}$ .

Cooling is only possible if the coolant volume inside the habitat stays below a given maximum fraction  $f_{C, \text{max}}$ ,  $V_{\text{abs}} + V_{\text{con,H}} < f_{C, \text{max}} V_{\text{H}}$ .

## B.6 Heat Emission

Emission pipes branch off the axial connector in a flat structure, returning with a U-turn to the back-flowing connection pipe. Their distances are such that their surface area equals the surface area of the radiator sheets ( $\pi D_{\text{em}} = 2d_{\text{em}}$  with the pipe diameter  $D_{\text{em}}$  and the pipe distance  $d_{\text{em}}$ ). Curved mirrors between the pipes permit a complete view of space (both sides of the radiator face a cold half-sphere).

Warm matter within the field of view provides a counter-radiation that hampers emission, e.g. parts of the energy collection system or the hull that are not shielded by mirrors. Also, the radiator is additionally heated if located near warm planetary bodies. Earth emits  $240 \frac{\text{W}}{\text{m}^2}$  of thermal radiation and reflects  $100 \frac{\text{W}}{\text{m}^2}$  of sunlight, providing a diffuse component of near-Earth sunlight. The latter can be reflected by a white coating, but the former is absorbed as the wavelength is similar to the radiator's emission, where the absorptance(=emissivity) is high. In LEO, Earth's thermal radiation comes from a third of the sky and thus averages to  $80 \frac{\text{W}}{\text{m}^2}$ . So the radiator absorbs about a quarter of its own emission level, reducing its effective emission by a quarter. This counter-radiation  $I_{\text{sky}}$  defines the sky temperature  $T_{\text{sky}} = \sqrt[4]{\frac{I_{\text{sky}}}{\sigma}}$ , which is the effective temperature of the surrounding space excluding the Sun (194K at  $80 \frac{\text{W}}{\text{m}^2}$ ). The Cosmic Microwave Background and other background radiation provide only a small counter-radiation (3K).

The temperature in the radiator decreases as heat is emitted (from  $T_{\text{R,in}}$  to  $T_{\text{R,out}}$ ). The effective temperature  $T_{\text{eff}}$  is here defined as the uniform temperature at which the radiator would emit the same power. Without internal friction and counter-radiation, the power is  $P_C + P_{\text{F,abs}} + P_{\text{F,con}} = \dot{m} \delta u_{\text{em}} = \epsilon \sigma A_{\parallel,0} T_{\text{eff}}^4$  with the emissivity  $\epsilon$ . The area without friction and counter-radiation  $A_{\parallel,0}$  depends on the coolant type:

- For **liquid** coolant the emitted power from a small area is  $\Delta P = \epsilon \sigma T^4 \Delta A$ . This power leads to a temperature decrease  $\Delta T = -\frac{\Delta P}{\dot{m} c_C}$ , so  $\Delta A = -\frac{\dot{m} c_C}{\epsilon \sigma} T^{-4} \Delta T$ . The

total emitting area that cools from the incoming temperature  $T_{R,\text{in}} = T_{C,\text{out}} + \Delta T_{\text{con}}$  to the desired outgoing temperature  $T_{R,\text{out}} = T_{C,\text{in}} - \Delta T_{\text{con}}$  is

$$A_{\parallel,0} = -\frac{\dot{m}c_C}{\epsilon\sigma} \int_{T_{R,\text{in}}}^{T_{R,\text{out}}} T^{-4} dT = \frac{\dot{m}c_C}{3\epsilon\sigma} (T_{R,\text{out}}^{-3} - T_{R,\text{in}}^{-3})$$

- For **vapor**, the latent heat from condensation is much higher than the sensible heat of vapor or liquid. Condensation occurs when the temperature drops below the boiling point, releasing the latent heat and keeping the temperature at the boiling point, which depends only on pressure. Thus if the density (and hence the pressure) is kept high by a decreasing cross section to compensate for condensed water, the pressure is nearly constant in the whole cooling system, and the temperature decrease in the radiator is small. The effective temperature is set by the required boiling point for heat absorption, which is assumed to be the minimum coolant temperature in the habitat:  $T_{\text{eff}} \approx T_{C,\text{in}}$ . This neglects two opposing effects: On the one hand, parts of the radiator that service warmer habitat regions can have higher temperature. On the other hand, pressure and thus temperature would be lower during emission than absorption to draw in vapor.
- For **air**, the assumption of nearly constant pressure leads to a decreasing density of gaseous water (by condensation), coupled to a decreasing dew point. The temperature follows the dew point due to the released latent heat, and the partial pressure of water is the saturation pressure  $p_{w,S}$ . The emitted power is  $\Delta P = -\dot{m}(c_C\Delta T + \phi\Delta h) = \epsilon\sigma T^4\Delta A$ , so

$$A_{\parallel,0} = -\frac{\dot{m}}{\epsilon\sigma} \int_{T_{R,\text{in}}}^{T_{R,\text{out}}} \left( c_C + \phi \frac{dh}{dT} \right) T^{-4} dT$$

Integration runs from incoming to outgoing temperatures. However, condensation starts only when the initial dew point  $T_{\text{dew}}$  is reached ( $\frac{dh}{dT} = 0$  above  $T_{\text{dew}}$ ). This temperature depends on the absolute humidity of the air leaving the habitat: Since  $p_{w,S}(T_{\text{dew}}) = p_w(T_{C,\text{out}})$ , it follows that  $e^{\frac{\phi}{R_w} \left( \frac{1}{273} - \frac{1}{T_{\text{dew}}} \right)} = h_{r,\text{out}} e^{\frac{\phi}{R_w} \left( \frac{1}{273} - \frac{1}{T_{C,\text{out}}} \right)}$  and  $\frac{\phi}{R_w} \left( \frac{1}{273} - \frac{1}{T_{\text{dew}}} \right) = \ln h_{r,\text{out}} + \frac{\phi}{R_w} \left( \frac{1}{273} - \frac{1}{T_{C,\text{out}}} \right)$ , so  $T_{\text{dew}} = \left( \frac{1}{T_{C,\text{out}}} - \frac{R_w}{\phi} \ln h_{r,\text{out}} \right)^{-1}$ .

While the first part of the integral is the same as for liquid, the last part is  $\int \frac{dh}{dT} T^{-4} dT = hT^{-4} + 4 \int hT^{-5} dT$ , using the chain rule of integration. Since the exact solution is not needed for the required level of accuracy, the integration  $\int_{T_{\text{dew}}}^{T_{R,\text{out}}} hT^{-5} dT$  is approximated as  $(T_{R,\text{out}} - T_{\text{dew}})h(T_{\text{av}})T_{\text{av}}^{-5}$  with the average temperature  $T_{\text{av}} = \frac{T_{\text{dew}} + T_{R,\text{out}}}{2}$ . This results in

$$A_{\parallel,0} \approx \frac{\dot{m}c_C}{3\epsilon\sigma} (T_{R,\text{out}}^{-3} - T_{R,\text{in}}^{-3}) + \frac{\dot{m}\phi}{\epsilon\sigma} (h(T_{\text{dew}})T_{\text{dew}}^{-4} - h(T_{R,\text{out}})T_{R,\text{out}}^{-4} + 4h(T_{\text{av}})T_{\text{av}}^{-5} (T_{\text{dew}} - T_{R,\text{out}}))$$

As seen in Sec. B.2, the specific humidity is  $h(T) \approx \frac{18p_{w,S}}{30p_A} = \frac{18p_{w,\text{tp}}}{30p_A} e^{\frac{\phi}{R_w} \left( \frac{1}{273} - \frac{1}{T} \right)}$ , as the relative humidity remains at 100% below the dew point.

The difference between coolant and surface temperature is neglected here because both liquid and condensation have a high transfer coefficient and the pipe walls a high conductivity. For air flow, the emission surface is considerably colder only at the beginning, before the onset of condensation.

It is assumed here that the pressure is roughly constant throughout the cooling system, although the flow is driven by pressure gradients. Pumps or fans would regularly increase the pressure to compensate for friction losses; pressure variations are therefore negligible. However, it is possible to actively compress air or vapor to increase the emitting temperature and to decrease the required cross section. The power needed for compression could be partly recovered by turbines when the coolant enters the habitat and expands. A problem would be condensation during the expansion phase - the required heat rejection could hamper the energy recovery. Although potentially promising, this probably makes it technically challenging. To avoid the extra complexity, it is not considered in the model.

Internal friction and counter-radiation both increase the necessary emitting area in proportion to the power increase. The friction power can be parameterized as  $P_{F,em} = \chi_{em}(P_C + P_{F,abs} + P_{F,con}) = \chi_{em}(1 + \chi_{abs} + \chi_{con}(1 + \chi_{abs}))P_C = \chi_{em}(1 + \chi_{abs})(1 + \chi_{con})P_C$ , thus increasing the emitted power by  $\chi_{em}$ , and counter-radiation adds a power of  $\epsilon\sigma A_{||,em}T_{sky}^4$ . The resulting heat rejection power is the sum of the power emitted from  $A_{||,0}$ , the friction power, and counter-radiation:  $\epsilon\sigma A_{||,em}T_{eff}^4 = (1 + \chi_{em})\epsilon\sigma A_{||,0}T_{eff}^4 + \epsilon\sigma A_{||,em}T_{sky}^4$ , so

$$A_{||,em} = (1 + \chi_{em}) \frac{T_{eff}^4}{T_{eff}^4 - T_{sky}^4} A_{||,0}$$

Since the radiator consists of two parts and emits on both sides, its length and surface area are related by  $A_{||,em} = 8R_{em}L_{em}$  with the radius (half-width)  $R_{em}$ . The shape is preferably quadratic ( $2R_{em} = L_{em}$ , so  $R_{em} = \frac{1}{4}\sqrt{A_{||,em}}$ ), but the radial extension can be limited to  $\kappa_{em}R_H$  for structural integrity (if the radiator co-rotates with the habitat).

Taking the surface density of the radiator hull  $\sigma_{em}$  into account, the mass of the radiator is  $M_{em} = \frac{2\dot{m}R_{em}}{w_{em}} + \sigma_{em}A_{||,em}$ .

## B.7 Friction

**Friction Power** Friction in the cooling system is an additional source of heat and sink of electricity. Pressure loss due to friction on a pipe wall is given by the Darcy-Weisbach equation  $\Delta p = \frac{\lambda\rho_C L w^2}{2D}$  with the (Darcy) friction factor  $\lambda$ , the pipe length  $L$  and diameter  $D$ .

The cross section (area perpendicular to the flow) is  $A_{\perp} = \frac{\pi}{4}D^2N_P$  with the number of pipes  $N_P$ , and the pipe wall area (parallel to the flow) is  $A_{||} = \pi D L N_P$ . Inserting the number of pipes from the first equation ( $N_P = \frac{4A_{\perp}}{\pi D^2}$ ) into the second equation yields  $D = \frac{4LA_{\perp}}{A_{||}}$ .

Maintaining the flow (compensating friction) requires a force of  $\Delta p A_{\perp}$ , applied by engines (pumps or fans) of efficiency  $\eta_F$ . The power is thus

$$P_F = \eta_F^{-1} \Delta p A_{\perp} w = \eta_F^{-1} \frac{\lambda\rho_C L w^2}{2D} \frac{A_{||} D}{4L} w = \frac{\lambda\rho_C}{8\eta_F} A_{||} w^3$$

**Friction Factor** The friction factor  $\lambda$  depends on the Reynolds number  $\Re = \frac{\rho_C w D}{\mu} = \frac{\rho_C w}{\mu} \frac{4LA_{\perp}}{A_{||}} = \frac{4L\dot{m}}{\mu A_{||}}$ . The dynamic viscosity  $\mu$  is about  $10^{-3} \frac{\text{kg}}{\text{ms}}$  for liquid water,  $1.8 \cdot 10^{-5} \frac{\text{kg}}{\text{ms}}$  for air, and  $8 \cdot 10^{-6} \frac{\text{kg}}{\text{ms}}$  for water vapor. These differences are almost exactly canceled by

the different densities (at 0.4bar air pressure), so for same velocity and pipe diameter the Reynolds numbers are similar for all coolants.

Below  $\Re \approx 2300$ , the flow is laminar (little movement perpendicular to the flow direction), and the friction factor is given by  $\lambda_{\text{lam}} = \frac{64}{\Re}$  (Hagen-Poiseuille).

At higher Reynolds numbers, the flow becomes turbulent, and for simplicity the Blasius relation is used:

$$\lambda_{\text{turb}} = 0.3164\Re^{-\frac{1}{4}}$$

The exact value depends on the Reynolds number as well as the ratio of surface roughness to diameter (Colebrook-White equation), and this approximation underestimates the friction factor at high  $\Re$  or high roughness. Therefore,  $\lambda$  is assumed to not decrease further than  $\lambda_{\text{min}} = 0.005$  (at  $\Re = 1.6 \cdot 10^7$ ), which prevents implausibly low friction factors at very high  $\Re$ . Still, a wind through the habitat would have a very rough lower surface and hence a higher friction factor, so the model computes the effective cross section if the surface was smooth (or only the part without obstacles).

The higher turbulent friction is assumed also for the transition regime, so when  $\Re$  climbs above 2300, the friction factor jumps from its laminar value of 0.028 to a turbulent value of 0.0457, then falls off slowly until reaching its assumed minimum value of 0.005. A turbulent flow distributes heat evenly over the cross section, which is advantageous in heat absorption and emission, while a laminar flow requires some periodic mixing for an efficient heat exchange.

**Velocity in Absorption and Emission** In heat absorption and emission,  $A_{\parallel}$  is fixed, and the number of pipes varies.  $\Re$  is independent of the flow velocity (the pipe diameter is proportional to cross section and hence inversely proportional to velocity, so  $wD$  is constant). With the length  $L = 2R_{\text{abs}}$ ,

$$\Re_{\text{abs}} = \frac{8R_{\text{abs}}\dot{m}}{\mu A_{\parallel,\text{abs}}}$$

If  $\Re_{\text{abs}} < 2300$ , or  $\frac{R_{\text{abs}}\dot{m}}{\mu} < 287.5A_{\parallel,\text{abs}}$ , the flow is laminar with friction power  $P_{\text{F,lam}} = \frac{8\rho_C}{\Re_{\text{abs}}\eta_{\text{F}}}A_{\parallel,\text{abs}}w_{\text{abs}}^3 = \frac{\mu\rho_C}{R_{\text{abs}}\dot{m}\eta_{\text{F}}}A_{\parallel,\text{abs}}^2w_{\text{abs}}^3$ , so the velocity is

$$w_{\text{abs,lam}} = \sqrt[3]{\frac{P_{\text{F,abs}}R_{\text{abs}}\dot{m}\eta_{\text{F}}}{\mu\rho_C A_{\parallel,\text{abs}}^2}}$$

For turbulent flow,  $P_{\text{F,turb}} = \frac{0.3164\rho_C}{8\eta_{\text{F}}}\Re_{\text{abs}}^{-\frac{1}{4}}A_{\parallel,\text{abs}}w_{\text{abs}}^3 = \frac{0.3164\rho_C}{8\eta_{\text{F}}}\left(\frac{\mu}{8R_{\text{abs}}\dot{m}}\right)^{\frac{1}{4}}A_{\parallel,\text{abs}}^{\frac{5}{4}}w_{\text{abs}}^3$ , and so

$$w_{\text{abs,turb}} = \sqrt[3]{\frac{8P_{\text{F,abs}}\eta_{\text{F}}}{0.3164\rho_C}\left(\frac{8R_{\text{abs}}\dot{m}}{\mu}\right)^{\frac{1}{4}}A_{\parallel,\text{abs}}^{-\frac{5}{4}}}$$

The same applies to heat emission (substituting  $_{\text{abs}}$  for  $_{\text{em}}$ ).

**Velocity in Connection** In heat connection, the number of pipes is fixed at  $N_{\text{P}} = 2$  (two radiators). Their length is  $L = 2L_{\text{eff}}$  (back and forth).

The connection pipe surface area  $A_{\parallel,\text{con}}$  depends on the velocity  $w_{\text{con}}$ : From  $D = \frac{4LA_{\perp}}{A_{\parallel}}$  and  $A_{\parallel} = \pi DLN_{\text{P}}$  it follows that

$$A_{\parallel,\text{con}} = \sqrt{4\pi L^2 A_{\perp,\text{con}} N_{\text{P}}} = 4L_{\text{eff}}\sqrt{\frac{2\pi\dot{m}}{\rho_C w_{\text{con}}}}$$

The Reynolds number is found from  $\Re = \frac{4L\dot{m}}{\mu A_{\parallel}}$ , so

$$\Re_{\text{con}} = \frac{1}{\mu} \sqrt{\frac{2\dot{m}\rho_C w_{\text{con}}}{\pi}}$$

Making use of the above expressions for  $\Re_{\text{con}}$  and  $A_{\parallel,\text{con}}$  and the assumed shape of the friction factor, powers and velocities for the three flow regimes can be derived:

- Below  $\Re = 2300$ , the *laminar* power is  $P_{\text{F,lam}} = \frac{64\rho_C}{8\eta_{\text{F}}\Re_{\text{con}}} A_{\parallel,\text{con}} w_{\text{con}}^3 = \frac{32\pi L_{\text{eff}}\mu}{\eta_{\text{F}}} w_{\text{con}}^2$ , and the flow velocity is  $w_{\text{con,lam}} = \sqrt{\frac{\eta_{\text{F}} P_{\text{F,lam}}}{32\pi L_{\text{eff}}\mu}}$
- Between 2300 and  $1.6 \cdot 10^7$ , the *turbulent* power can then be expressed as  $P_{\text{F,lowturb}} = \frac{0.3164\rho_C}{8\eta_{\text{F}}} \Re_{\text{con}}^{-\frac{1}{4}} A_{\parallel,\text{con}} w_{\text{con}}^3 = \frac{0.3164}{\eta_{\text{F}}} 2^{-\frac{5}{8}} \pi^{\frac{5}{8}} L_{\text{eff}} (\rho_C \dot{m})^{\frac{3}{8}} \mu^{\frac{1}{4}} w_{\text{con}}^{\frac{19}{8}}$ . The turbulent velocity follows to be  $w_{\text{con,lowturb}} = \left( \frac{2^{\frac{5}{8}} \eta_{\text{F}} P_{\text{F,con}}}{0.3164 \pi^{\frac{5}{8}} L_{\text{eff}} (\rho_C \dot{m})^{\frac{3}{8}} \mu^{\frac{1}{4}}} \right)^{\frac{8}{19}}$ .
- At high  $\Re$ , the *constant- $\lambda$*  assumption gives  $P_{\text{F,highturb}} = \frac{\lambda_{\text{min}}\rho_C}{8\eta_{\text{F}}} A_{\parallel,\text{con}} w_{\text{con}}^3 = \frac{\lambda_{\text{min}} L_{\text{eff}}}{2\eta_{\text{F}}} \sqrt{2\pi\dot{m}\rho_C} w_{\text{con}}^{\frac{5}{2}}$ , so the velocity is  $w_{\text{con,highturb}} = \left( \frac{2\eta_{\text{F}} P_{\text{F,con}}}{\lambda_{\text{min}} L_{\text{eff}} \sqrt{2\pi\dot{m}\rho_C}} \right)^{\frac{2}{5}}$ .

The transition velocities are found from the corresponding Reynolds numbers. The transition between laminar and turbulent flow occurs at  $w_{\text{lam-turb}} = (2300\mu)^2 \frac{\pi}{2\rho_C \dot{m}} = \frac{8.31 \cdot 10^6 \mu^2}{\rho_C \dot{m}}$ , and between low and high turbulence at  $w_{\text{low-high}} = \frac{\pi\mu^2}{2\rho_C \dot{m}} \left( \frac{0.3164}{\lambda_{\text{min}}} \right)^8 = \frac{4.04 \cdot 10^{14} \mu^2}{\rho_C \dot{m}}$ . The corresponding powers provide boundaries between the flow regimes. The power gap in the laminar-turbulent transition would produce  $w_{\text{lam-turb}}$ , but this velocity is (for liquid water)  $10^{-2} \frac{\text{m}}{\text{s}}$  divided by the mass flow in  $\frac{\text{kg}}{\text{s}}$ , so the laminar region requires extremely low velocities and is therefore not implemented (the large diameter typically leads to high Reynolds numbers). The high-Reynolds region with constant  $\lambda$  is reached when  $P_{\text{F,con}} > P_{\text{F,low-high}} = \frac{\lambda_{\text{min}} L_{\text{eff}}}{2\eta_{\text{F}}} \sqrt{2\pi\dot{m}\rho_C} w_{\text{low-high}}^{\frac{5}{2}} = \frac{0.3164^{20} \pi^3 \mu^5 L_{\text{eff}}}{8\lambda_{\text{min}}^{19} \eta_{\text{F}}} (\dot{m}\rho_C)^{-2} = \frac{3.918 \cdot 10^{-10} \mu^5 L_{\text{eff}}}{\lambda_{\text{min}}^{19} \eta_{\text{F}} \dot{m}^2 \rho_C^2}$ .

## B.8 Minimizing Mass-to-Power

The mass of a space habitat is a rough estimate of the effort it takes to build it (although each component and material would have its own cost-to-mass ratio). Thus, the cooling system would be designed with a focus on minimizing its mass (under more complex constraints such as reliability and redundancy). This section provides an approximate analytical determination of the friction fractions  $\chi$  that minimize the cooling system mass for a given habitat power. The parameter  $b_{\text{opt}}$  determines if friction fractions are optimized (solutions to  $\chi$  from this section) or fixed (solutions to  $w$  from Sec. B.7). The optimized friction fractions are scaled so their sum stays below a given maximum  $\chi_{\text{max}}$ . In the case of air coolant, optimization is not applied to heat absorption; instead the flow velocity is given by the available windy air volume.

Note that the approximate optimum requires a simplification of the mass equations, as well as start values of the  $\chi$  parameters (given by the input parameters  $\chi_{\text{abs,st}}$ ,  $\chi_{\text{con,st}}$ , and  $\chi_{\text{em,st}}$ ). This is because iterations are avoided so there is no interaction between the components during optimization. Also, an optimum cooling network would consist of meandering pipes and many connections between parts of habitat and radiator, instead of the assumed single axial pipe. Furthermore, technical considerations such as risk and



wearing mitigation could favor lower flow velocities. Nevertheless, the solution does a rough optimization of flow velocities to better scale the model in a large power range.

If  $\chi$  and  $\lambda$  are constant and  $A_{\parallel}$  proportional to the habitat power, flow velocity and cross section per habitat power are (roughly) constant. Since the lengths increase with size, however, larger habitats have more cooling mass per power, and would benefit from spending more power on friction. The minimal mass can be found by varying the friction powers, i.e. finding optimum values of  $\chi$ . The cross section and hence the coolant mass decreases with higher velocity, while mass flow, radiator and PV surface increase with the higher thermal and electrical load.

The heat dissipation mass  $M_{\text{HD}}$  is the sum of coolant and surface masses of the three components, plus the mass of electricity generation for cooling  $M_{\text{E,c}} = P_{\text{F}}\mu_{\text{E}}$  (where  $\mu_{\text{E}} = \frac{\sigma_{\text{E}}}{\eta_{\text{E}}I}$  is the mass per electric power):

$$\begin{aligned} M_{\text{HD}} &= M_{\text{C,abs}} + M_{\text{C,con}} + M_{\text{C,em}} + M_{\text{S,abs}} + M_{\text{S,con}} + M_{\text{S,em}} + M_{\text{E,c}} \\ &= 2\dot{m} \left( \frac{R_{\text{abs}}}{w_{\text{abs}}} + \frac{L_{\text{abs}} + L_{\text{em}} + 2L_{\text{con}}}{2w_{\text{con}}} + \frac{R_{\text{em}}}{w_{\text{em}}} \right) + \sigma_{\text{abs}}A_{\parallel,\text{abs}} + \sigma_{\text{con}}A_{\parallel,\text{con}} + \sigma_{\text{em}}A_{\parallel,\text{em}} + P_{\text{F}}\mu_{\text{E}} \end{aligned}$$

The advantage of larger  $\chi$  is lower coolant mass;  $M_{\text{C}} \sim \frac{1}{w} \sim \chi^{-\beta}$  with the exponent  $\beta = \frac{1}{3}$  in heat absorption and emission,  $\frac{8}{19}$  in the connection at lower  $\Re$  and  $\frac{2}{5}$  at higher  $\Re$ . The disadvantage is a linear increase in electricity consumption and heat dissipation, so in PV and radiator surfaces as well as in radiator length. The latter is proportional to the coolant mass in the outside connection pipes. Additionally, friction in heat absorption linearly increases the mass flow and hence the total coolant mass. With a linear term  $M_{\text{lin}}$  (penalty for higher friction), the mass equation can be written as  $M_{\text{HD}} = M_{\text{lin,st}} \frac{1+\chi}{1+\chi_{\text{st}}} + M_{\text{C,st}} \left( \frac{\chi}{\chi_{\text{st}}} \right)^{-\beta} + \text{const}$  for each of the three friction fractions ( $M_{\text{lin,st}}$  and  $M_{\text{C,st}}$  are evaluated using their start values). The general linear mass is  $M_{\text{lin,gen}} = M_{\text{C,con,out}} + M_{\text{S,em}} + \mu_{\text{E}}(1 + \chi_{\text{abs,st}})(1 + \chi_{\text{con,st}})(1 + \chi_{\text{em,st}})P_{\text{C}}$  (adopted for emission and connection). In heat absorption, it is  $M_{\text{lin,abs}} = M_{\text{C,abs}} + M_{\text{C,con}} + M_{\text{C,em}} + M_{\text{lin,gen}}$ .

The optimum friction fractions  $\chi_{\text{abs,opt}}$ ,  $\chi_{\text{con,opt}}$ , and  $\chi_{\text{em,opt}}$  should yield a minimum mass, i.e. the derivation with respect to  $\chi_{\text{abs}}$ ,  $\chi_{\text{con}}$ , and  $\chi_{\text{em}}$ , respectively, is 0:  $\frac{M_{\text{lin,st}}}{1+\chi_{\text{st}}} - \beta M_{\text{C,st}} \chi_{\text{st}}^{\beta} \chi_{\text{opt}}^{-\beta-1} = 0$ , so the minimum is at

$$\chi_{\text{opt}} = \left( \frac{\beta(1 + \chi_{\text{st}})\chi_{\text{st}}^{\beta} M_{\text{C,st}}}{M_{\text{lin,st}}} \right)^{\frac{1}{\beta+1}}$$

It follows that the three optima are at

$$\begin{aligned} \chi_{\text{abs,opt}} &= \left( \frac{(1 + \chi_{\text{abs,st}})\chi_{\text{abs,st}}^{\frac{1}{3}} M_{\text{C,abs}}}{3(M_{\text{C,abs}} + M_{\text{C,con}} + M_{\text{C,em}} + M_{\text{lin,gen}})} \right)^{\frac{3}{4}} \\ \chi_{\text{con,opt}} &= \left( \frac{\beta_{\text{con}}(1 + \chi_{\text{con,st}})\chi_{\text{con,st}}^{\beta_{\text{con}}} M_{\text{C,con}}}{M_{\text{lin,gen}}} \right)^{\frac{1}{\beta_{\text{con}}+1}} \end{aligned}$$

where  $\beta_{\text{con}} = \frac{8}{19}$  if  $P_{\text{F,con}}$  is below the transition power between the two turbulent regions ( $\chi_{\text{con,st}}(1 + \chi_{\text{abs,st}})P_{\text{C}} < \frac{2.05}{\eta_{\text{F}}\dot{m}^2}\mu_{\text{C}}^5 L_{\text{eff}}$ ). It is  $\frac{2}{5}$  in the constant- $\lambda$  region.

$$\chi_{\text{em,opt}} = \left( \frac{(1 + \chi_{\text{em,st}})\chi_{\text{em,st}}^{\frac{1}{3}}M_{\text{C,em}}}{3M_{\text{lin,gen}}} \right)^{\frac{3}{4}}$$

## References

- [1] J. R. Brophy, L. Friedman, and F. Culick. “Asteroid retrieval feasibility”. In: *2012 IEEE Aerospace Conference*. 2012, pp. 1–16. DOI: 10.1109/AERO.2012.6187031. URL: [https://www.kiss.caltech.edu/final\\_reports/Asteroid\\_final\\_report.pdf](https://www.kiss.caltech.edu/final_reports/Asteroid_final_report.pdf).
- [2] Muhao Chen et al. “Review of space habitat designs for long term space explorations”. In: *Progress in Aerospace Sciences* 122 (Apr. 2021), p. 100692. DOI: 10.1016/j.paerosci.2020.100692.
- [3] Al Globus, Ankur Bajoria, and Joe Straut. “The Kalpana One Orbital Space Settlement Revised”. In: (Dec. 2010). URL: <http://alglobus.net/NASAwork/papers/2007KalpanaOne.pdf>.
- [4] Al Globus, Stephen Covey, and Daniel Faber. “Space Settlement: An Easier Way”. In: *NSS Space Settlement Journal* (July 2017). DOI: 10.13140/RG.2.2.12690.35523. URL: <https://space.nss.org/wp-content/uploads/NSS-JOURNAL-Space-Settlement-An-Easier-Way.pdf>.
- [5] Al Globus and Joe Strout. “Orbital Space Settlement Radiation Shielding”. In: *NSS Space Settlement Journal* (Apr. 2017). DOI: 10.13140/RG.2.2.27789.84966. URL: <https://space.nss.org/wp-content/uploads/NSS-JOURNAL-Orbital-Space-Settlement-Radiation-Shielding.pdf>.
- [6] Pekka Janhunen. “Shielded dumbbell L5 settlement”. In: *NSS Space Settlement Journal* (Apr. 2020). URL: <https://space.nss.org/wp-content/uploads/NSS-JOURNAL-Shielded-Dumbbell-L5-Settlement.pdf>.
- [7] Richard D. Johnson. *Space Settlements: A Design Study*. NASA SP-413. Vol. 413. 1977. URL: [https://space.nss.org/settlement/nasa/75SummerStudy/Table\\_of\\_Contents1.html](https://space.nss.org/settlement/nasa/75SummerStudy/Table_of_Contents1.html).
- [8] Gerard K. O’Neill. *The High Frontier: Human Colonies in Space*. William Morrow and Co, 1976. ISBN: 0688031331.